Modelling of random vehicle loading histories for fatigue analysis

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Abstract: The concise description of one dimensional vehicle loading histories for fatigue analysis using stochastic process theory is presented in this study. The load history is considered to have stationary random and nonstationary mean content. The stationary variations are modelled by an Autoregressive Moving Average (ARMA) model, while a Fourier series is used to model the variation of the mean. Due to the use of random phase angles in the Fourier series an ensemble of mean variations is obtained. The methods of nonparametric statistics are used to determine the success of the modelling of nonstationarity. Justification of the method is obtained through comparison of rainflow cycle distributions and resulting fatigue lives of original and simulated loadings. Due to the relatively small number of Fourier coefficients needed together with the use of ARMA models, a concise description of complex loadings is achieved. The overall frequency content and sequential information of the load history is statistically preserved. An ensemble of load histories can be constructed on-line with minimal computer storage capacity as used in testing equipment.

Keywords: ARMA models, correlations, ensemble, fatigue analysis, nonstationary, random histories, variability, vehicle loadings.

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1 Introduction

Vehicle loading histories are often lengthy and random in nature. For successful design against fatigue failure, simulation studies such as the Monte Carlo method and laboratory testing are undertaken. An accurate and concise description of the loading, therefore, is desirable. The methods of modelling irregular fatigue loading histories can be divided into two groups, namely counting methods and methods based on correlation theory (Bílý and Bukoveczky, 1976).

First, model-free techniques evaluate the record via a count. These methods consider only the extreme values which reduces the required storage by discarding all intermediate points. They work well for fatigue loading histories in the absence of creep effects, because only the extremes induce fatigue damage, while intermediate points are irrelevant. In this class, most commonly used are the Rainflow matrix method (Endo *et al.*, 1974) and the To-From matrix method (Haibach *et al.*, 1976).

On the other hand, there are descriptions of random loadings based on correlation theory. For these techniques, the model becomes a substitute for the data, which leads to a concise description with few parameters. A method proposed by Yang (1972) represents the data by its power spectral density, i.e. the frequency domain description of the autocorrelation of the original data. The Markov method, as described by Cacko *et al.* (1988), falls in this category, as do a more general class of time series called Autoregressive Moving Average (ARMA) models.

Another publication by the authors of this study (Thangjitham *et al.*, 1994) discusses in detail the use of ARMA models for stationary fatigue loading histories. Traditionally, ARMA models have been used in the areas of earthquake (Kozin, 1988), wind (Li and Kareem, 1990) and ocean engineering (Spanos, 1983) to model random load histories. An article reviewing ARMA models for Monte Carlo studies is due to Spanos and Mignolet (1989).

Random processes can be analysed either in the time or frequency domain. Techniques in the time domain are employed because of their efficiency in simulating loadings. Furthermore, random processes can be classified into two categories, stationary and nonstationary. Nonstationary processes have certain characteristics such as mean or variance that change over time. The modelling of nonstationarity is important because many real loadings are nonstationary.

The history to be modelled in this study, taken from a ground vehicle travelling on a rough road, is considered to consist of a slowly varying process, the nonstationary mean variation, and a fast varying process, the stationary random variation. To account for such mean variation in an accurate but concise manner, Fourier series are employed for their versatility with respect to describing loadings and their ability to be extended to a stochastic process. ARMA models are used for their efficiency in describing stationary random processes. Finally, an ensemble of loadings is obtained from the observation of a single record, because both mean and random variation are presented by stochastic processes.

2 Nonstationary random fatigue load model

The model developed to describe fatigue random load histories is applicable to both stationary and nonstationary cases, and the mean variation can be modelled as being either deterministic or stochastic.

2.1 Assumptions

The time history is a superposition of a zero-mean stationary random process and events which affect the variation of the mean. These two components contribute distinctly to the power spectral density (PSD) of the combined process. The mean variation is of slowly varying nature and contributes only to the low frequency range of the PSD. The stationary random variation, however, may contribute to the PSD at any frequency.

For the case studied, the random loading, Figure 1(a), represents actual data of the strain response at a given point of a vehicle travelling over a rough road. The irregular road profile induces strain, which is of stationary random nature, while manoeuvres induce nonstationary variations in strain with respect to mean. The assumption of manoeuvres being of slow varying nature is justified through the analysis of actual driving behaviour (McLean and Hoffmann, 1971).

2.2 *The time series representation*

To represent the vehicle loading history with nonstationary mean variation, the following model then is employed:

$$x(t) = m(t) + n(t) \tag{1}$$

where $x(t)$ represents the underlying history, $n(t)$ is a zero-mean stationary random process, and $m(t)$ is the nonstationary variation in the mean value. In this study, the variation of the mean value will be treated either deterministic or stochastic.

To minimize the number of parameters necessary to characterize the mean variation in a deterministic manner a truncated Fourier series is used such that

$$m(t) = \tfrac{1}{2}a_0 + \sum_{k=1}^{M_\mu}\left[a_k \cos\left(\omega_0 k \Delta tt\right) + b_k \sin\left(\omega_0 k \Delta tt\right)\right] \tag{2}$$

where Δt is the length of the sample interval, $\omega_0 = 2\pi / \Delta tN$ is the fundamental frequency, M_μ and N are the number of terms in the truncated Fourier series and the total number of sample points of the history, respectively, and a_k and b_k are the discrete Fourier coefficients. For the case of $M_\mu \ll (N/2\text{-}1)$, $m(t)$ will be approximating the low frequency content of $x(t)$, i.e., its mean variation. The value of M_μ is found such that the difference between the original history and truncated Fourier series yields a zero mean process, $n(t)$.

$$n(t) = x(t) - m(t) \tag{3}$$

To find the parameter of M_μ one proposed method is due to Buxbaum and Zaschel (1977), who analyse the dynamic system to decide which part of the response spectrum is due to stationary and nonstationary loadings. Filtering in the frequency domain allows one to separate the two components. This, however, is often difficult as information regarding the dynamic system characteristics and the actual input spectrum are seldom available. Therefore, to determine whether the series $n(t)$ is indeed stationary with respect to its mean value, the methods of nonparametric statistics are used.

The remaining stationary component, $n(t)$, then can be represented by an ARMA model of appropriate order. The selection of the proper ARMA model is made such that a *parsimonious* model, i.e. a model that uses the smallest necessary number of parameters to approximately account for correlations, is chosen to present the random series.

2.3 *ARMA models*

There are two components of an ARMA model: (1) the autoregressive part, and (2) the moving average part. The autoregressive part represents the dependence of the output variable (observed variable) on its own past. For example, a second-order autoregressive model, denoted AR(2), is defined as follows:

$$n(t) - \phi_1 n(t-1) - \phi_2 n(t-1) = a(t) \tag{4}$$

where $n(t)$ is the variable under observation (output variable), $a(t)$ is the shock or driving noise (input variable), and ϕ_1 and ϕ_2 are the autoregressive parameters. The parameter t refers to discrete points in time, $t \in (1, 2, 3, \ldots)$ and is a nominal one, since it is only used to characterize the time dependence in the data, which is considered valid irrespective of any particular value of t. In the above model, the input process $a(t)$ is assumed to be an independently and identically distributed random process with zero mean and constant variance σ_a^2. That is, $a(t)$ itself is considered to be non-autoregressive is considered to be non-autoregressive.

The moving average part of the ARMA models represents the dependence of the output process on the past values of the input process. For example, the following is a pure moving average model of order two, denoted as MA(2):

$$n(t) = a(t) - \theta_1 a(t-1) - \theta_2 a(t-2) \tag{5}$$

where θ_1 and θ_2 are the moving average parameters. The above model is a second-order moving average model, because a current value of the output depends on the past two values of the input process.

The full ARMA model is formed by a combination of the autoregressive and moving average parts. For example, the following is an example of a second-order autoregressive and first order moving average model, denoted as ARMA(2,1):

$$n(t) - \phi_1 n(t-1) - \phi_2 n(t-2) = a(t) - \theta_1 a(t-1) \tag{6}$$

where the above equation represents second-order dynamics, with two being the order of the autoregressive part. Typically, any physical process which is a result of a second-order governing differential equation in time can be represented by the above equation.

High order ARMA models can be used to represent more complex dynamics. The ARMA(p,q) model, therefore, is expressed as:

$$\begin{aligned} &n(t) - \phi_1 n(t-1) - \phi_2 n(t-2) - \ldots - \phi_p n(t-p) \\ &= a(t) - \theta_1 a(t-1) - \theta_2 a(t-2) - \ldots - \theta_q a(t-q) \end{aligned} \tag{7}$$

where the autoregressive parameters, $\phi_1; i = 1, 2, \ldots p$, and the moving average parameters, $\theta_1; i = 1, 2, \ldots q$, are estimated from the observed data using standard statistical procedures (Box and Jenkins, 1976).

The estimation of ARMA parameters is based on a method of moments procedure. The autocorrelation function of the input record, $n(t)$ is obtained; this is referred to as the target spectrum, where the autocorrelation function of a process is a measure of correlation among data points as a function of their separation, or *lag time*. The ARMA parameters are estimated such that the autocorrelation function of the respective ARMA model is as close to the target spectrum as possible.

In ARMA modelling the proper model order (p,q) is found to be the one that transforms the observed data, $n(t)$, to an uncorrelated series, $a(t)$. Pandit (1973) and Akaike (1974) developed criteria based on rigorous statistical tests to determine the appropriate model order. Unfortunately, for a large number of data points, these criteria tend to be too restrictive, i.e. they demand models of very large order.

To overcome this problem, a criterion was developed which selects among a group of possible models the one which would lead to a fatigue life similar to the one of the observed series while using the smallest number of parameters necessary. This ensures that all dynamic characteristics, relevant for fatigue load simulation are present in a chosen ARMA model (Leser, 1993).

3 Nonparametric statistics

To determine whether a sequence of observations is of random nature, statistical tests can be performed. If no information about the distribution function of the sequence is available, a nonparametric test is desired because no assumptions regarding distributions are necessary. In nonparametric inference, the methods are based only on the relative occurrence of an underlying event. Therefore, information or assumptions regarding the population are not necessary to assess whether a sequence is of random nature or contains deterministic trends (Gibbons, 1971).

Three nonparametric tests are presented here to determine the stationarity of $n(t)$. In general, given a time series of length N, one divides this series into N_I intervals each containing N_P points, such that $N = N_I \times N_P$ The mean value is calculated for each interval. The interval means, μ_i, are considered as the sequence of observations to be tested for randomness.

The first test is based on the variables R_T, the total number of runs. A single run is defined as successive observations of the interval mean below or above the median and is completed when two succeeding observations are separated by the median. This is the best known and most general run test. The test is focused on a single quantity, the median, and gives a general measure of randomness or lack thereof without identifying a trend or cyclical pattern. Hald (1952) derives the mean and variance for the expected number of runs for a true random sequence. For the case where the total number of runs, R_T, is larger than twenty, the distribution of R_T is approximately normal. For a sequence of length N_I, the expected value and variance for R_T then are defined as (Hald, 1952)

$$E[R_T] = \mu_{R_T} = \frac{N_I + 2}{2} \tag{8}$$

$$E\left[\left(R_T - \mu_{R_T}\right)^2\right] = \mathrm{var}[R_T] = \sigma^2_{R_T} = \frac{N_I}{4}\left(1 - \frac{1}{N_I - 1}\right) \tag{9}$$

Confidence limits for the expected number of runs can be established. A hypothesis test is based on the comparison of the observed number of runs, r_T, and the theoretically expected number of runs, μ_{R_T}. Confidence limits at level α are defined as (Hald, 1952)

$$\left(r_T - z_{\alpha/2}\sigma_{R_T}\right) \le \mu_{R_T} \le \left(r_T + z_{\alpha/2}\sigma_{R_T}\right) \tag{10}$$

where $z_{\alpha/2}$ is defined such that

$$1 - \alpha/2 = \int_{z_{\alpha/2}}^{\infty} e^{-\frac{1}{2}x^2}\,dx \tag{11}$$

If the observed number of runs, r_T, falls inside the confidence limits, Equation (10), for a chosen confidence level, usually $\alpha = 0.95$, the hypothesis of the sequence being random is accepted, while for the case where r_T falls outside these limits the observed sequence must be considered deterministic.

The second run test is based on the variable for the length of the longest run, K. In a random sequence of length N_I, the longest of the runs described above follows the relation (Hald, 1952)

$$N_I \cong K - \frac{1}{K+1} - \frac{K+2}{2} K! \log_e\left(P\left(-z_{\alpha/2} < Z < z_{\alpha/2}\right)\right) \tag{12}$$

and

$$P\left(-z_{\alpha/2} < Z < z_{\alpha/2}\right) = \int_{-z_{\alpha/2}}^{z_{\alpha/2}} e^{-\frac{1}{2}x^2} dx \tag{13}$$

Since Equation (12) cannot be solved directly for K, the test is indirect. The hypothesis test at confidence level α for the observed longest run requires checking whether the series is sufficiently long to admit a run of observed length, k, in a random sequence of length N_I. This test is best suited for identifying trends in a sequence.

Finally, tests based on the number of runs up and down, R_{UD}, provide another measure of randomness of a sequence of interval means. The magnitude of each observation is compared with that of the immediately preceding observation. If the next element is larger, a run up is started, and if smaller, a run down. A decision concerning randomness is then based on the number of these runs, while the length is not considered. This test traces the whole sequence of observations relative to each other, in contrast to the test based on the total number of runs. Therefore, a periodic fluctuation (cyclic content) of the observed sequence can be detected through the number or runs up and down. A hypothesis test can be derived for measuring whether the observed number of runs, r_{UD}, significantly deviates from the expected value for a random sequence, $\mu_{R_{UD}}$. The expected value and variance of the number of runs up and down for a random sequence of length N_I are (Hald, 1952)

$$E[R_{UD}] = \mu_{R_{UD}} = \frac{1}{3}(2N-1)) \tag{14}$$

$$E\left[\left(R_{UD} - \mu_{R_{UD}}\right)^2\right] = \text{var}[R_{UD}] = \sigma^2_{R_{UD}} = \frac{1}{90}\left(16N_1 - 29\right) \tag{15}$$

Confidence limits at level α are defined as

$$\left(r_{UD} - z_{\alpha/2}\sigma_{R_{UD}}\right) < \mu_{R_{UD}} < \left(r_{UD} + z_{\alpha/2}\sigma_{R_{UD}}\right) \tag{16}$$

Each run test detects a certain form of deviation from the case of a random sequence of interval means. Too few runs, runs of excessive length, or too many runs can be used as statistical criteria for the rejection of the hypothesis of randomness of the sequence of μ_i. Therefore, all tests should be considered, and need to be passed successfully, for a sequence (in this study, $n(t)$) to be considered stationary.

4 Ensemble generation

Successful load modelling for the purpose of realistic fatigue testing asks for an ensemble of load histories where each realization (history) is representative of the actual loading. In practice, it is often difficult to obtain sufficient information about the ensemble. It is common to have only a limited number of representative records for a particular loading situation. Therefore it is desired, given a single loading, to obtain a large number of realizations which are not identical, but contain variations which have statistical characteristics identical to the original history.

The proposed model of Equation (3) allows for an extension from a single observed record to an ensemble in all its components. The ARMA models employed to account for the stationary variations of the loads are of stochastic nature. It is not a single time series that is embodied in a particular set of ARMA parameters, but a random process, which for each generation yields a different realization with identical stochastic characteristics, but a different sequence and values of relative extrema.

The ensemble of mean variations can be obtained using a method introduced by Rice (1944). It is shown that a time series, $y(t)$, of a random signal can be described by its discrete Fourier transform

$$y(t)=\frac{1}{2}a_0+\sum_{k=0}^{\frac{N-1}{2}}\left[a_k\cos\left(\omega_0 k\Delta tt-\alpha_k\right)+b_k\sin\left(\omega_0 k\Delta tt-\beta_k\right)\right] \tag{17}$$

where a_k, b_k, ω_0 , Δt and N were defined above and α_k and β_k are two random phase angles distributed uniformly over the interval $(0, 2\pi)$.

This representations yield an ergodic random process, i.e. a process that is stationary, such that an average taken over time is identical to an average taken across the ensemble of histories. Histories with a different variation for each simulation of Equation (17) are obtained, yet the overall characteristics, such as the frequency content and variance, are preserved. A stochastic description of the process $y(t)$ is obtained.

This formulation of a random process can be used to derive an ensemble of mean variations by modifying the mean description in Equation (2) in two ways. First, similarly to Equation (17) random phase angles are added to the mean description of Equation (2). This leads to mean representations whose spectral content will be preserved, but the sequence of events, i.e. the occurrence of a relative or absolute maximum, will be different for each simulation. While these records are statistically identical it might be desirable to obtain an ensemble of mean variations where each realization will have a prescribed similarity (correlation) to the original mean variation. This would yield more realistic load simulations as the sequence of major events, such as relative extrema, can be preserved to any desired degree.

This objective leads to a second modification of Equation (2). The prescribed correlation between original mean content and the simulated mean of the model is obtained through limiting the number of terms which carry a random phase angle in Equation (17). Instead of α_k and β_k being random phase angles for all k, α_k and β_k will be restricted such that

$$\alpha_k,\beta_k=\begin{cases}U & \text{for} \quad k\in\left(N_z+1,M_\mu\right)\\ 0 & \text{for} \quad k\in\left(0,N_z\right)\end{cases} \tag{18}$$

and the mean description becomes

$$m_{N_z}(t) = \frac{1}{2}a_0 + \sum_{k=0}^{M_\mu}\left[a_k \cos(\omega_0 k\Delta tt - \alpha_k) + b_k \sin(\omega_0 k\Delta tt - \beta_k)\right] \tag{19}$$

where N_z indicates the number of terms with zero phase angles and M_μ, and a_k and b_k were defined in Equation (2), and U indicates the uniform distribution defined on the interval $(0,2\pi)$. This definition of α_k and β_k allows for a systematic characterization of the correlation between the deterministic mean representation, $m(t)$, and the ensemble of random mean variations, $m_{N_z}(t)$.

From a strict theoretical standpoint, the random process of Equation (19) with the definition of α_k and β_k of Equation (18) yields a non-ergodic process. This is due to the fact that the process contains deterministic components, i.e. terms with zero phase angles. However, the proposed method of using a limited number of random phase angles conserves the frequency content of the deterministic mean variation.

In order to measure the correlation between the deterministic mean, $m(t)$, and a simulated mean, $m_{n_z}(t)$, a correlation coefficient is obtained via the customary definition

$$\rho_{N_z} = \frac{E\left[m(t)m_{N_z}(t)\right]}{\sqrt{E\left[m^2(t)m_{N_z}^2(t)\right]}} \tag{20}$$

This definition implies that for $\rho_{N_z=0}$ deterministic mean and simulated mean are uncorrelated, while for $\rho_{N_z=1}$ they are identical. In general, the larger the value of ρ_{N_z} the closer the resemblance between deterministic and simulated mean.

5 Results and discussion

In this paper, fatigue life calculations (MTS Systems Corporation, 1991) are presented for SAE 1045 steel, subjected to both original and reconstructed strain histories. Table 1 shows the material constants (Kurath *et al.*, 1989). The local strain approach is employed in life calculations. Consequently, the analysis predicts the initiation of easily detectable *engineering size* cracks.

Table 1 Material properties for SAE 1045 steel.

Modulus of elasticity, E(MPa)	202000.0
Fatigue strength coefficient, σ'_f (MPa)	948.0
Cyclic strength coefficient, H' (MPa)	1258.0
Cyclic strain hardening exponent, n'	0.208
Fatigue strength exponent, b	–0.092
Fatigue ductility coefficient, ε'_f	0.260
Fatigue ductility exponent, c	–0.445

For the purpose of life analysis, a typical history of nonstationary strain gauge data taken from a particular component of a ground vehicle travelling on a rough road is chosen as the random fatigue load history in this study, Figure 1(a). The history is considered to be observed at a critical location such as the notch root in an engineering component. This history, containing 10 240 points, is from now on referred to as a *block*. Without loss of generality, the load history has been normalized such that its overall mean value is zero and its dimensionless root mean square (RMS) value is equal to unity. Furthermore, the sampling rate is assumed to have been one point per second. The normalized minimum and maximum are found to be –3.7850 and 2.8663, respectively, while the skewness coefficient is computed as –0.3074, which indicates that more data points fall below the zero-mean than above it.

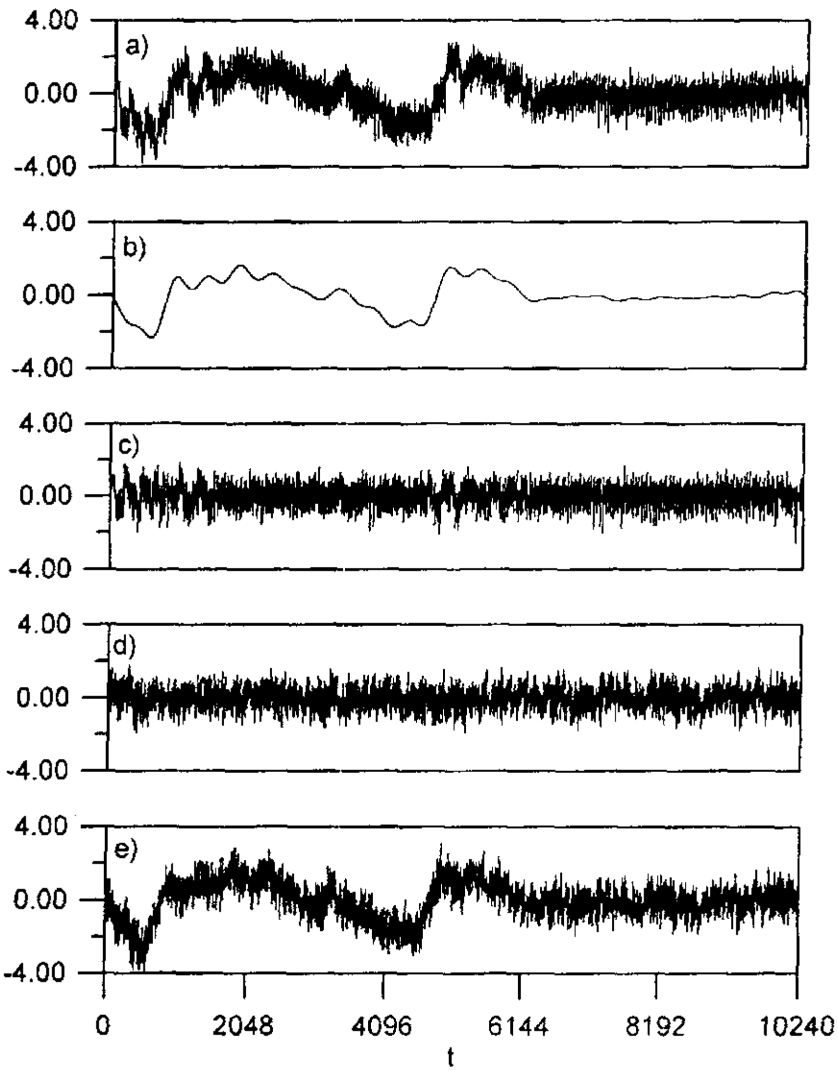

Figure 1 Times series plots for (a) original record, (b) deterministic mean presentation, (c) mean removed record, (d) ARMA model simulation, and (e) reconstruction.

According to the employed model of Equation (1), the history is decomposed into its two components, $m(t)$ and $n(t)$. To model the variation of the mean in a deterministic way, $m(t)$, various Fourier series with increasing numbers of terms are formed, giving the tentative mean descriptions. The difference, $n(t)$, of the original record and each mean description, one of which is shown in Figure 1(b), is obtained. These differences, such as Figure 1(c), are then analysed for deviations from being a zero-mean process. The best mean description is chosen as the one that renders $n(t)$ stationary using the Fourier series with the least number of terms.

Next, the mean-removed record, $n(t)$, is divided into N_I intervals, for each of which the interval mean is determined. These intervals need to be long enough to give reliable estimates for the mean, yet short enough to be able to detect variations in the mean of the whole record. Furthermore, for proper statistical analysis, it is desirable to treat the estimates of the interval means as if they were uncorrelated to each other. No common rule has been established in the literature as to what this interval length should be. One method is provided via the autocorrelation function of the signal (Bendat and Piersol, 1986). The autocorrelation for the series in question vanishes for a lag time of approximately 60 points and more. This indicates that data points separated by more than 60 points are not correlated.

Another argument to support the choice of interval size can be made using inference methods from classical statistics. To establish the necessary number of data points to estimate the interval mean, μ_i, a Student's t-test can be used (Miller and Freund, 1977)

$$\left(\bar{x}_i - t_{\alpha,N_P-1}\frac{S_i}{\sqrt{N_P}}\right) \le \mu_i \le \left(\bar{x}_i + t_{\alpha,N_P-1}\frac{S_i}{\sqrt{N_P}}\right) \tag{21}$$

where $\bar{x}_i$ is the estimated interval mean and S_i is the square root of the unknown interval variance and t_{α,N_P-1} indicate Student's t-distribution with α level of confidence and $N_P - 1$ degrees of freedom. An expression indicating the relative maximum error in estimation of μ_1 can be derived as

$$\text{relative maximum error} = \left|\frac{\bar{x}_i - \mu_i}{S_i}\right| \le \frac{t_{\alpha,N_P-1}}{\sqrt{N_P}} \tag{22}$$

For a chosen value of $\alpha = 0.9$ and an acceptable relative maximum error of 20% of μ_i, the required number of sample points is $N_P = 64$. This leads to the choice of using $N_I = 160$ intervals, each containing $N_P = 64$ points. However, since this choice is not unique, tests are also performed for $N_I = 128$ and $N_P = 80$.

Run tests based on the total number of runs, the number of runs up and down, and the length of the longest run, are performed on the sequence of interval means calculated from $n(t)$. Table 2. This assures that a variety of deviations from the expected random behaviour of this sequence can be detected.

Table 2 *Results of run tests for mean modelling for different values of M_μ (italics = failure of test, bold passed all tests for given N_I).*

M_μ	$68<r_T<93$	$95<r_{UD}<116$	$N_I=160$	$53<r_T76$	$75<r_{UD}<94$	$N_I=128$
	r_T	r_{UD}	k	r_T	r_{UD}	k
5	*27*	*78*	*8*	*19*	*58*	*8*
10	*54*	*88*	*7*	*43*	*64*	5
15	71	*88*	*7*	**60**	**78**	**5**
20	72	*92*	6	*51*	76	4
25	**83**	**98**	4	**75**	**86**	**3**
30	*94*	*100*	3	*84*	*86*	4
35	*105*	106	3	*87*	92	3
40	*96*	102	3	*86*	87	3
45	*115*	*117*	3	*92*	*97*	3
50	*121*	*123*	3	*106*	*111*	3

For the case where $N_P=64$, the 95% ($\alpha=0.95$) confidence limits for the total number of runs, μ_{R_T}, are ($68<\mu_{R_T}<93$), while the number of runs up and down, $\mu_{R_{UD}}$, covers the range ($95<\mu_{R_{UD}}<116$). Similarly for the case where $N_P=80$, the 95% confidence limits for the total number of runs, $\mu_{R_{UT}}$, span over the range ($53<\mu_{R_{UT}}<76$), while the number of runs up and down covers the range ($75<\mu_{R_{UT}}$,94). The length of the longest admissible run, K, according to Equation (12) for a random sequence of length 160 is 6, while for a sequence of length 128, the maximum length is 5.

For the case with $N_P=64$, the only value for which all run tests are passed is $M_\mu=25$. For the run tests based on $N_P=80$, all run test are passed by values for M_μ of 15 and 25. Therefore, a total number of $M_\mu=25$ Fourier series coefficients is deemed appropriate for a sufficient mean description to render the remaining signal stationary with respect to its mean value. See also Figure 1(b) for the deterministic mean model, $m(t)$, and Figure 1(c) for the mean removed record, $n(t)$.

The stationary sequence will be presented by an ARMA model, Figure 1(d). An ARMA(10,0) model, i.e. ten autoregressive parameters and zero moving average parameters, was deemed suitable to represent $n(t)$ accurately and concisely. A complete simulation according to Equation (1) using the stationary record obtained from the selected ARMA model and the chosen mean representation is shown in Figure 1(e).

An ensemble of mean variations was obtained according to Equation (19). Depending on the parameter N_z, correlations as defined in Equation (20) between the deterministic mean variation, $m(t)$, and 64 simulations were calculated to obtain a reliable estimate. Figure 2 shows these results. Figures 3(b-e) show a set of four mean variations of different correlation with the deterministic mean. These records were obtained for correlation values of $\rho_{N_z}=(0.95, 0.74, 0.56)$ corresponding to values for $N_z=(18, 6, 4)$. Finally, a random phase angle was added to each term, $N_z=0$, so that the deterministic mean and simulated mean are uncorrelated. Figures 3(f-g) show these simulations. These simulated records are drastically different from the original. This method, therefore, allows one to obtain mean simulations with any desired closeness to the deterministic mean.

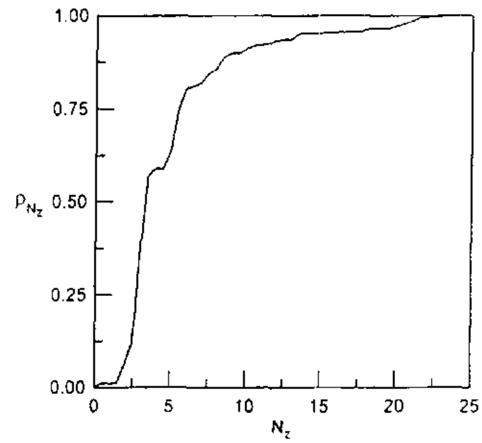

Figure 2 Correlation ρ_{N_z}, between deterministic and ensemble mean for different values N_z.

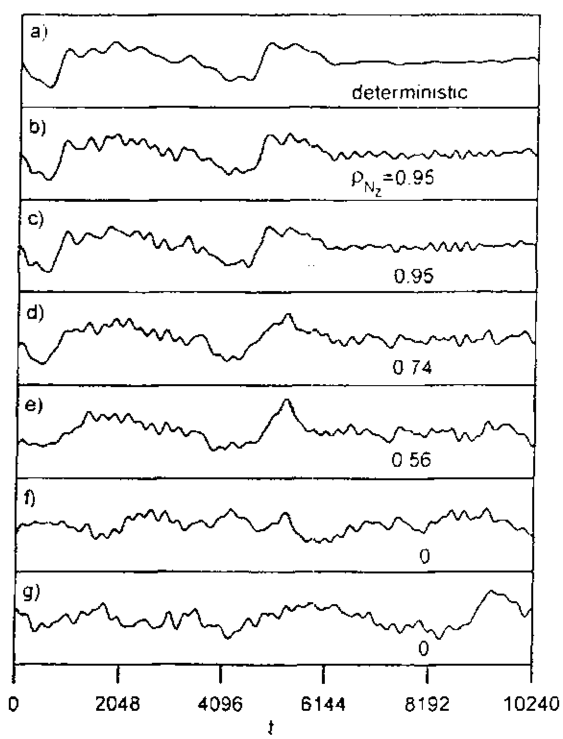

Figure 3 Time series plots for (a) deterministic mean record and mean ensemble presentations correlated to the original, (b) correlated at $\rho_{N_z} = 95\%$, (c) correlated at $\rho_{N_z} = 95\%$, (d) correlated at $\rho_{N_z} = 74\%$, (e) correlated at $\rho_{N_z} = 56\%$, (f) correlated at $\rho_{N_z} = 0\%$, and (g) another realization correlated at $\rho_{N_z} = 0\%$.

In the following analysis, the variations of fatigue life as a function of the RMS strain level are considered (Dowling, 1972, and Dowling *et al.*, 1977, and MTS Systems Corporation, 1991), the so called *strain life curve*. Consequently, the strain (load) histories corresponding to various RMS levels are required. This can be accomplished by simply multiplying the normalized load history and its regeneration with a given RMS value. Strain life curves were obtained for 64 simulations and averaged. Figure 4 shows the comparison for various RMS strain levels of original record and average of simulation. The closeness of original and simulation for a wide range of strain levels renders this simulation successful for both low cycle and high cycle fatigue, i.e. short and long life applications can be simulated with the above regeneration.

Rainflow cycle distributions and the associated damage distributions are shown in Figure 5 for one typical RMS strain level. The visual comparison of rainflow cycle distributions for the original and reconstructed record indicates good agreement for the various ranges and means represented in the history. This indicates that the simulation was successful in faithfully reconstructing both the large number of cycles with small range as well as approximately the few cycles with large range.

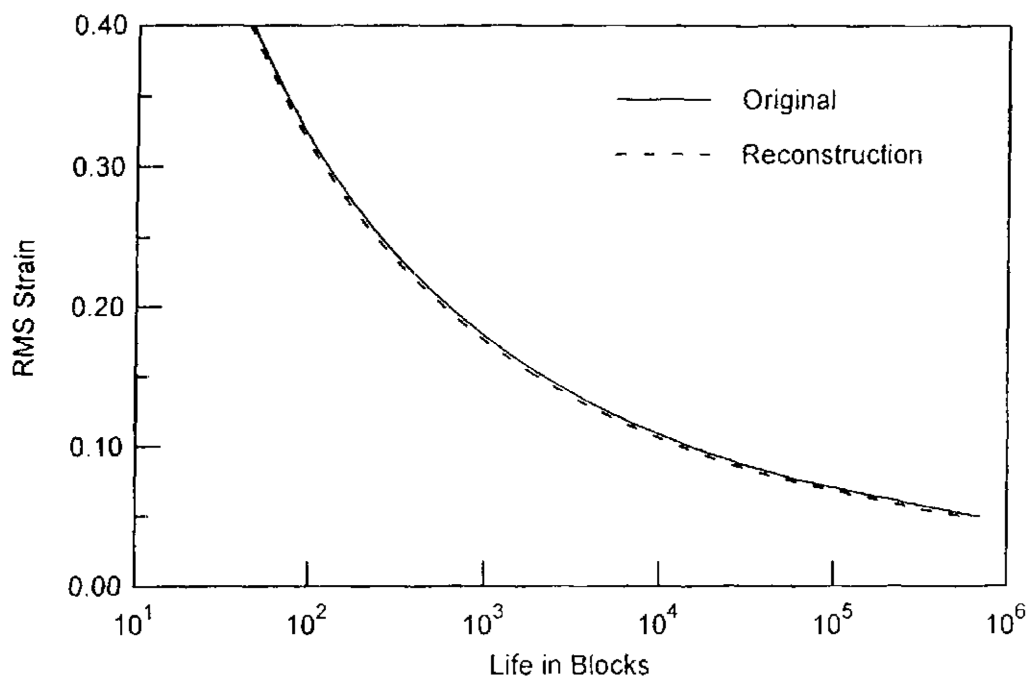

Figure 4 Plot of strain life curves calculated for original record and average of 64 reconstructions.

To measure the variability in fatigue life, 64 simulations were performed where both mean and random content were generated independently. The fatigue life was calculated for each simulation to obtain the mean, standard deviation, and coefficient of variation (ratio of standard deviation and mean). The variability of fatigue life for different correlation values, ρ_{N_t}, of the mean realization is shown in Figure 6.

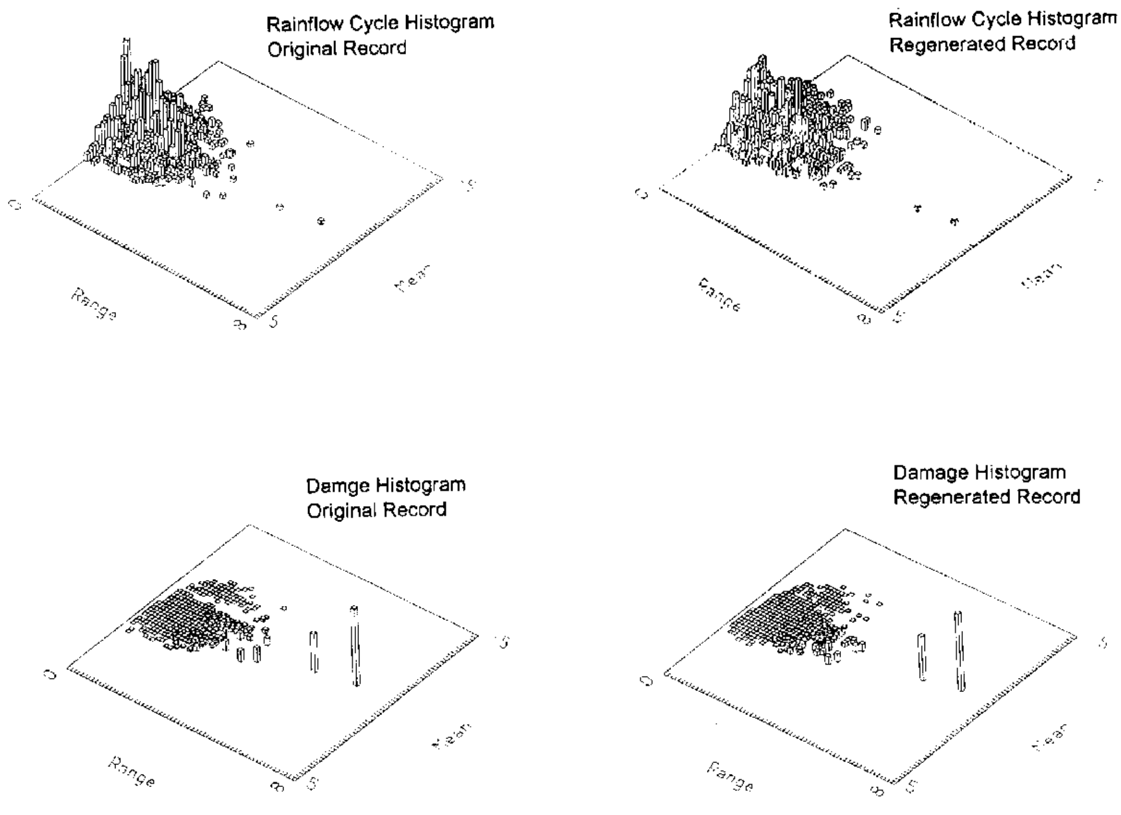

Figure 5 Plot of rainflow and damage histograms for original and a regenerated record at RMS strain level 0.1%.

For the case of smaller values of ρ_{N_z} the variability in life is larger due to the contribution of the larger variations in mean. For the limiting case of $\rho_{N_z} = 1$, i.e. where the deterministic mean was used, the variability is smallest. The variability in life for all cases of ρ_{N_z} is larger for smaller value of RMS strain level. This is due to the fact that as the strain level decreases the large number of rainflow cycles with small range contribute less to the overall damage (see also Figure 5). Therefore, only a few large range cycles contribute to fatigue damage, consequently the variability is larger.

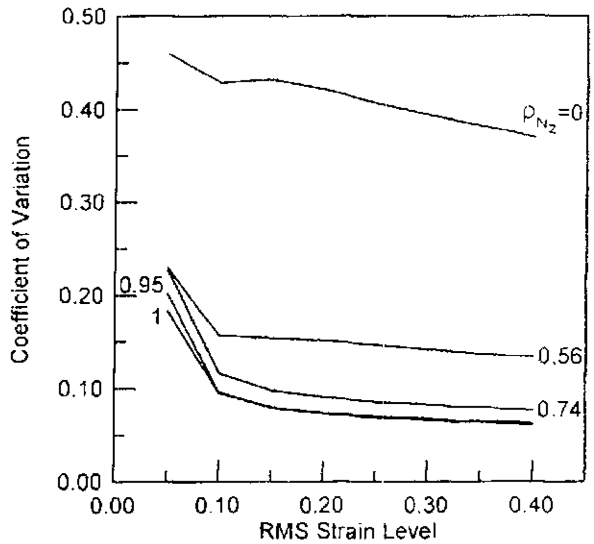

Figure 6 Plot of variability of fatigue life versus RMS strain level for mean realization of various correlations, ρ_{N_z}, to deterministic mean.

6 Summary

A vehicle load history model for fatigue analysis and testing is represented. Both nonstationarities with respect to mean and stationary random content are concisely modelled. The mean variation can be treated in either a deterministic or a stochastic manner, where the stochastic case is adaptive in that desired correlations between simulated and original record can be obtained. The generation of an ensemble is provided for by each component in the proposed model. The frequency content of the original loading is preserved in the simulation, allowing the application to realistic dynamic and multiaxial loadings. Moreover, the sequence of events, which can be a factor in fatigue life, is preserved in a statistical sense. Infinitely long records can be simulated with minimal computer storage in real time. The description uses fewer parameters than any competing method of vehicle loading description.

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