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Modeling of non-stationary variance in vehicle loading histories for fatigue analysis

Christoph Leser*, Surot Thangjitham† and Norman E. Dowling†

The concise description of one dimensional vehicle loading histories for fatigue analysis using stochastic process theory is presented in this study. The load history is considered to have stationary random and nonstationary variance content. The stationary variations are modeled by an Autoregressive Moving Average (ARMA) model, while a Fourier series is used to model the estimated variation of the variance. Due to the use of random phase angles in the Fourier series an ensemble of variance variations can be obtained. Justification of the method is obtained through comparison of power spectral densities, time histories and resulting fatigue lives of original and simulated loadings. Due to the relatively small number of Fourier coefficients needed together with the use of ARMA models, a concise description of complex loadings is achieved. The overall frequency content and sequential information of the load history is statistically preserved. An ensemble of load histories can be constructed on-line with minimal computer storage capacity as used in testing equipment.

Introduction

Vehicle loading histories are often lengthy and of random nature. For successful design against fatigue failure, simulation studies such as the Monte Carlo method and laboratory testing are undertaken. An accurate and concise description of the loading, therefore, is desirable. The methods of modeling irregular fatigue loading histories can be divided into two groups, namely counting methods and methods based on correlation theory, Bílý and Bukoveczky (1).

First, model free techniques evaluate the record via a count. These methods consider only the extreme values which reduces the required storage by discarding all intermediate points. They work well for fatigue loading histories in the absence of creep effects, because only the extremes induce fatigue damage, while intermediate points are irrelevant. In this class, most commonly used are the Rainflow matrix method, Endo et al. (2), and the To-From matrix method, Haibach et al. (3)

On the other hand, there are descriptions of random loadings based on correlation theory. For these techniques, the model becomes a substitute for the data, which leads to a concise description with few parameters. A method proposed by Yang (4)

* MTS Systems Corporation, 14000 Technology Drive, Eden Prairie, MN 55344, USA
† Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA

represents the data by its power spectral density, i.e., the frequency domain description of the autocorrelation of the original data. The Markov method as described by Cacko et al. (5) falls in this category, as do a more general class of time series called Autoregressive Moving Average (ARMA) models.

A previous publication by the author of this study, Leser (6), discusses in detail the use of ARMA models for stationary fatigue loading histories. Traditionally, ARMA models have been used in the areas of earthquake-, wind-, and ocean-engineering to model random load histories.

Random processes can be analyzed either in the time or frequency domain. Techniques in the time domain are employed here because of their efficiency in simulating loadings. Furthermore, random processes can be classified into two categories, stationary and nonstationary. Nonstationary processes have certain characteristics such as mean or variance that change over time. The modeling of nonstationarity is important because many real loadings are of nonstationary nature.

The history to be modeled in this study, taken from a ground vehicle traveling on a rough road, is considered to consist of a slowly varying process, the nonstationary variance variation, and a fast varying process, the stationary random variation. An acceleration record of a ground vehicle travelling over a rough road (e.g. cobblestone) at different speeds would generate a record of such type. To account for such variance variation in an accurate but concise manner, Fourier series are employed for their versatility with respect to describing loadings and their ability to be extended to a stochastic process. ARMA models are used for their efficiency in describing stationary random processes. Finally, an ensemble of loadings can be obtained from the observation of a single record, because both variance and random variation are presented by stochastic processes.

Time Series Model

It is assumed that the time history under investigation is a superposition of a zero-mean stationary random process and events which affect the variation of the variance.

To represent the random fatigue loading the following model then is employed:

$$x_t = s_t \cdot n_t \tag{1}$$

where x_t represents the underlying history, s_t is the scaling function accounting for the variation in variance, and n_t a zero-mean stationary random process. The following sections will show how the components of Eq. 1 are modeled. It is understood that the parameter *t* refers to discrete points in time, as this study is concerned with the modeling of sampled time series.

The scaling function, s_t, is defined as the function that renders the quotient x_t / s_t stationary with respect to variance. This is equivalent to saying that s_t is defined as an estimator of the standard deviation of x_t. In order to estimate the standard deviation of x_t a procedure as shown by Nau et al. (7) is employed.

The time series, x_t, sampled at discrete equally spaced intervals, a simple estimate, $\tilde{\sigma}_t^2$, for the true variance, σ_t^2, is obtained via a moving window such as

$$\tilde{\sigma}_t^2 = \sum_{j=0}^{n} w_{j-\frac{n}{2}} x_{t+j-\frac{n}{2}}^2 \tag{2}$$

where n is the width of the window and the window weights, w_j, are such that

$$\sum_{j=0}^{n} w_{j-\frac{n}{2}} = 1 \qquad w_j \geq 0 \tag{3}$$

To determine an appropriate size, n, of the window, inference methods from classical statistics can be used. Via a Chi Square test a confidence interval can be constructed, Miller and Freund (8), such as

$$\left(\frac{\chi_{n-1,1-\alpha}^2}{n-1}\right) \leq \frac{\tilde{\sigma}_t^2}{\sigma_t^2} \leq \left(\frac{\chi_{n-1,\alpha}^2}{n-1}\right) \tag{4}$$

where σ_t^2 is the value of the true variance, $\tilde{\sigma}_t^2$ is the estimated variance, and $\chi_{n-1,\alpha}^2$ indicates the Chi Square distribution with $(n-1)$ degrees of freedom at confidence level α. For an acceptable relative maximum error of 25% the following must hold $0.75 \leq \tilde{\sigma}_t^2 / \sigma_t^2 \leq 1.25$. These bounds, with a chosen value of $\alpha = 0.9$, require a minimum number of $n = 96$, therefore, for numerical simplicity a value of $n = 100$ is chosen to estimate the true variance, σ_t^2.

The simplest weighting function is the rectangular one, i.e. $w_j = 1/(n+1)$. However, it is usually preferable to use a more gradually varying window, such that neighboring points have a stronger influence on the estimate of the variance than points that are further away from the current observation. Reference (7) refers to the use of a cosine bell shaped window, while in this study, for simplicity, a triangular window is introduced such that

$$w_j = \begin{cases} \dfrac{4}{n^2} j & \text{for} \quad 0 \leq j \leq \dfrac{n}{2} \\ -\dfrac{4}{n^2} j + \dfrac{4}{n} & \text{for} \quad \dfrac{n}{2} \leq j \leq n \end{cases} \tag{5}$$

It is shown in (7) that the estimate of the variance via Eq. 2 tends to be biased in a systematic way. Peak values in variance will be underestimated, while estimated troughs will be larger than the corresponding true values. In order to obtain a more accurate estimate a correction could be introduced to account for this known deviation. However, as a concise, and therefore only approximate, description of the variance is desired, no further refinement is performed.

$\tilde{\sigma}_t$ then gives an estimation of the standard deviation of x_t and can, therefore, be used to derive an estimate for the scaling function, s_t.

The next step is to concisely represent the estimated standard deviation, $\tilde{\sigma}_t$. The fact that $\tilde{\sigma}_t$ is not evenly distributed makes it difficult to postulate models that would describe it. Therefore, a transformation due to Box and Cox (9) is commonly used to enhance symmetry

$$\tilde{\sigma}_t^{BC} = \begin{cases} \tilde{\sigma}_t^{\lambda} & \text{for} \quad \lambda \neq 0 \\ \log \tilde{\sigma}_t & \text{for} \quad \lambda = 0 \end{cases} \tag{6}$$

where $\tilde{\sigma}_t^{BC}$ indicates the Box Cox transform of $\tilde{\sigma}_t$. The parameter λ is chosen such that the transformed series has zero skewness, i.e. it becomes symmetrically distributed about it's mean, in order to facilitate modeling by a harmonic function.

In this study, the scaling function, s_t, is a truncated Fourier series

$$s_t^{BC} = \frac{1}{2} c_0 + \sum_{k=1}^{M} \left[c_k \cos(\omega_0 kt\Delta t) + d_k \sin(\omega_0 kt\Delta t) \right] \tag{8}$$

where as before Δt is the length of the sample interval, $\omega_0 = 2\pi/(N\Delta t)$ is the fundamental frequency, M and N are the number of terms in the truncated Fourier series and the total number of sample points of the history, respectively, and c_k and d_k are the discrete Fourier coefficients. For the limiting case where $M = (N/2-1)$, $s_t^{BC} = \tilde{\sigma}_t^{BC}$, while for $M < (N/2-1)$, s_t^{BC} is an approximation of $\tilde{\sigma}_t^{BC}$ leading to s_t as a suitable scaling function. The value of M is found in this study such that s_t^{BC} and $\tilde{\sigma}_t^{BC}$ have a prescribed correlation coefficient of $\rho^M = 0.95$. M is much smaller than $(N/2-1)$, since the variation in variance has been calculated via an average and is therefore of slowly varying nature.

Ensemble Variance

The Fourier series to describe the scaling function, s_t, will be augmented by random phase angles, which in turn will be restricted according to the desired correlations between deterministic and stochastic scaling functions.

Given the Box-Cox transformation of the deterministic scaling function of Eq. (6) an ensemble of Box-Cox transformed scaling functions becomes

$$s_t^{N_Z} = \frac{1}{2}c_0 + \sum_{k=1}^{M}\left[c_k \cos(\omega_0 kt\Delta t - \gamma_k) + d_k \sin(\omega_0 kt\Delta t - \delta_k)\right] \tag{9}$$

with the random phase angles such that

$$\gamma_k, \delta_k = \begin{cases} U & \text{for} \quad k \in (N_Z + 1, M) \\ 0 & \text{for} \quad k \in (0, N_Z) \end{cases} \tag{10}$$

where N_Z indicates the number of terms with zero phase angles and U indicates the uniform distribution on the interval $(0, 2\pi)$. Depending on the value N_Z ensembles can be generated that vary in their deviation from the initially estimated record of Eq.(8).

ARMA Models

There are two components of an ARMA model: the autoregressive part and the moving average part. The autoregressive part represents the dependence of the output variable (observed variable) on its own past. The moving average part represents the dependence of the output process on the past values of the input process.

The full ARMA model is formed by a combination of the autoregressive and moving average parts:

$$n_t - \phi_1 n_{t-1} - \phi_2 n_{t-2} - \ldots - \phi_p n_{t-p} = a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \tag{7}$$

where the autoregressive parameters, ϕ_i; $i = 1, 2, \ldots p$, and the moving average parameters, θ_i; $i = 1, 2, \ldots q$, are estimated from the observed data using standard statistical procedures, Box and Jenkins (10). The input process, a_t, is assumed to be an independently and identically distributed random process with zero mean and constant variance σ_a^2. That is, a_t itself is considered to be non-autoregressive.

Fatigue Life Calculation

The fatigue life for original record and various reconstructions are reported as a function of the RMS strain level, Dowling (11). It is therefore assumed that the records under investigation represent local strains at a critical location of structure or component. A rainflow cycle count is performed and the Palmgren-Miner rule applied to calculate the fatigue life per history. By multiplying the records with a given RMS level and calculating the fatigue life using the material constants of a mild steel (SAE 1045) strain life curves are obtained.

Results

A typical history of nonstationary acceleration data is chosen in this study, Fig. 1. This history, contains 10240 points, its power spectral density is shown in Fig. 2.

According to the employed model of Eq. (1), the history is decomposed into its two components, the scaling function, s_t, and the stationary random part, n_t, where it is assumed that the mean is constant and zero. To model the scaling function, s_t, an estimate of the standard deviation of the time series, $\tilde{\sigma}_t$, is obtained according to Eq. (2), and shown in Fig. 1. In order to concisely represent $\tilde{\sigma}_t$ the Box-Cox transformation is performed. The optimal transformation parameter is found to be $\lambda = 0.275$, and the transformed variable, $\tilde{\sigma}_t^{BC}$, is shown in Fig. 1. A number of Fourier series with an increasing number of terms are formed according to Eq. (8), giving the tentative scaling functions. The series, s_t^{BC}, with the fewest number of terms that is correlated at 95% to $\tilde{\sigma}_t^{BC}$ has $M = 50$ terms and is also shown in Fig. 1. The inverse Box-Cox transformation of s_t^{BC}, s_t, is the scaling function used to render the original series stationary with respect to variance, see Fig. 1. The stationary series is the quotient of x_t and s_t, Fig. 1.

This stationary series will be presented by an ARMA model. Parameters for a number of ARMA models are estimated and the correlation coefficients between power spectra of these ARMA models and the spectrum of the stationary series are calculated (not shown here). Seeking models which have correlations of $\rho^{(p,q)}$ greater or equal than 0.8, 0.9, and 0.95 leads to the following choices of respective minimum order models: ARMA(2,0), ARMA(2,1), and ARMA(6,0).

Reconstructions are formed by multiplying the ARMA model time series and the Fourier series. The ARMA(0,0) model yields the limiting case with the shortest fatigue life, ARMA(1,0) the limiting case with the longest fatigue life and ARMA (6,0) is the case with the fatigue life closest to the original loading, Fig. 3. Both time history (Fig. 1) and power spectral density (Fig. 2) of the complete reconstruction are shown and agree well with the original record.

Summary

A vehicle load history model for fatigue analysis and testing is represented. Nonstationarities with respect to variance and stationary random content are concisely modeled. The variance variation can be treated in either a deterministic or a stochastic manner, where the stochastic case is adaptive in that desired correlations between simulated and original record can be obtained. The generation of an ensemble is provided for by each component in the proposed model. The frequency content of the original loading is preserved in the simulation, allowing the application to realistic dynamic and the extension to multiaxial loadings. Moreover, the sequence of events, which can be a factor in fatigue life, is preserved in a statistical sense. Infinitely long records can be simulated with minimal computer storage in real time. The description uses fewer parameters than any competing method of vehicle loading description.

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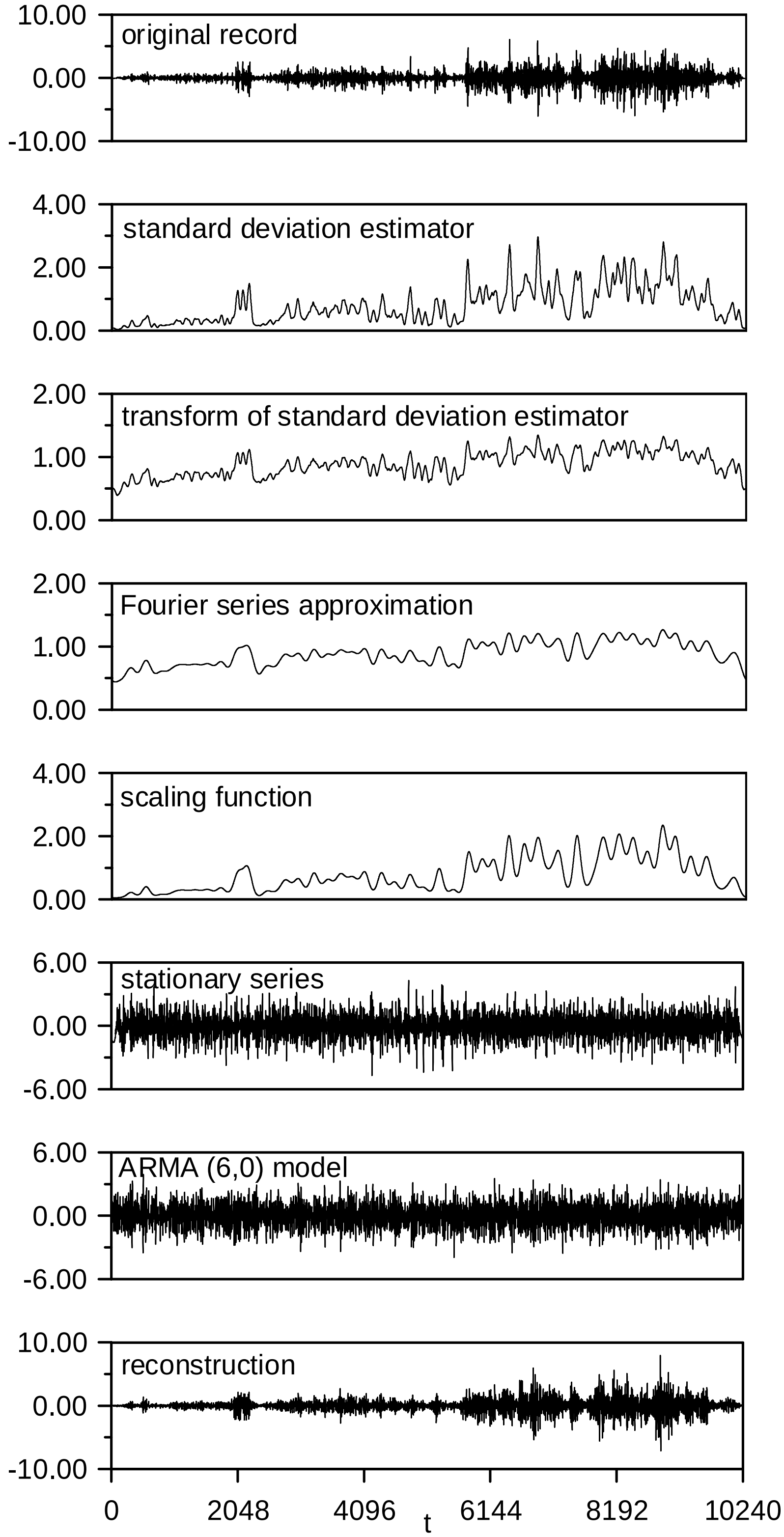

Figure 1 Original history, modeling procedure, and reconstruction.

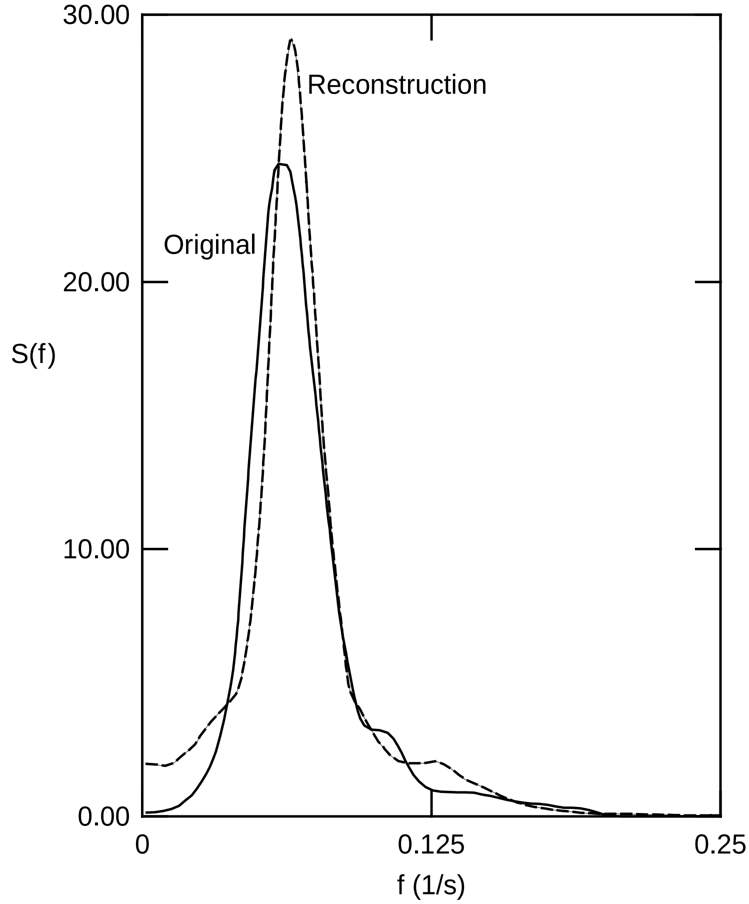

Figure 2 Power spectral density of the original and reconstructed history.

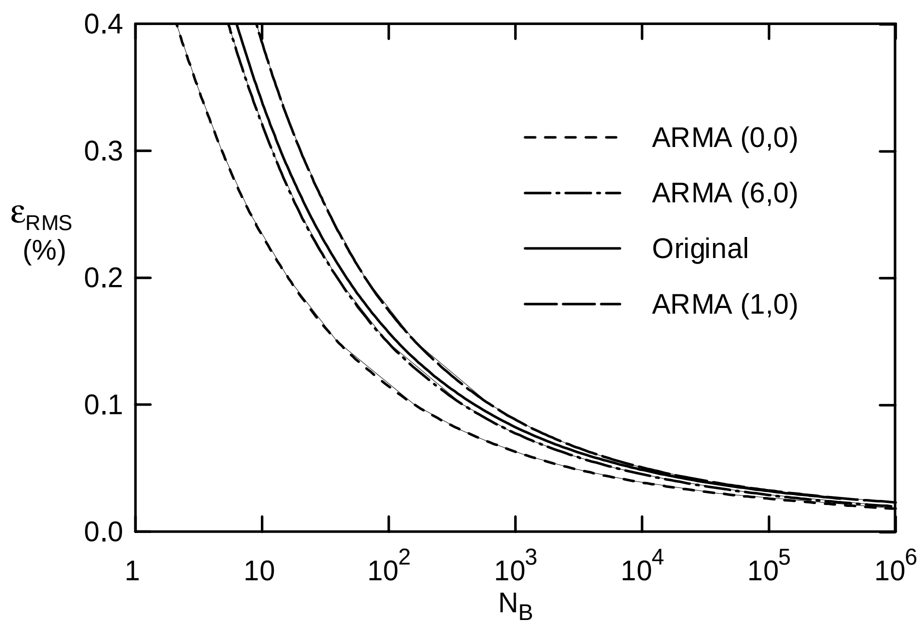

Figure 3 Strain-life curves for original and reconstructed histories.