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Abstract

A concise method for modeling nonstationary fatigue loading histories is presented. The mininum number of model parameters is achieved by fitting the variations in mean and variance by a truncated Fourier series. An autoregressive moving average (ARMA) model is used to describe the stationary component. Justification of the method is made by comparing fatigue relevant parameters obtained when subjected to the original and reconstructed histories. In spite of a relatively small number of parameters required, the model is shown to give good results that fall within the bounds predicted by the orginal history.

Introduction

Engineering structures are commonly subjected to complex fluctuating-load environments. For example, ground vehicles are subjected to vibratory loads that result from road surface irregularities, while pressure vessels and pipes are usually under combined time-varying mechanical and thermal loads. More often than not, these loadings are of a random nature and cannot be described by a simple mathematical expression [1,2]. Despite this fact, many engineering designs have been based on a deterministic concept that assumes complete information on the histories of applied loads is known in advance. This is due mostly to the lack of practical and uniform approaches for analyzing and characterizing random load histories, as well as to the inherent difficulties in theoretical calculations and laboratory verifications of the design objectives under such a random loading environment.

Due to the fact that general fatigue loading histories are lengthy and random in nature, the development of an accurate but nevertheless concise method for describing such histories is deemed necessary. The method is required not only to be able to preserve all fatigue relevant events but also to contain a minimum number of model parameters. The methods of modeling random fatigue loading histories are divided into two categories [3]–one that models only the extreme events [4-6] and the other that models the complete history [7-10].

Modeling the complete fatigue loading history requires a continuous dynamic model. The model parameters are obtained by time series analysis of the given history. Time series analysis can be performed in both the frequency and time domains. In the frequency domain, the statistical dependence in the data is implicitly recognized by treating the data as a combination of sine and cosine waves of various frequencies and amplitudes. This can be accomplished by performing spectral analysis to obtain the corresponding *power spectral density* (PSD) function.

When time series analysis is performed in the time domain, the statistical dependence in the data is explicitly recognized in the model representation. The dependence in the data is represented by the autocorrelation function which is defined for a given lag-time. Generally, the autocorrelation function goes to zero if the lag-time is very large. That is, the observation at the current time is almost independent of the observations from the distant past.

In this study, the time domain approach will be utilized to model nonstationary random fatigue loading histories. It is assumed that a typical *block* of the original history, $D(t)$, is made available in terms of N observations measured at a constant time interval Δ. This implies that both the mean and standard deviation of the random load history are periodic functions with period $T = N\Delta$.

Time Series Representation of Fatigue Loading Histories

To provide a general analytical model, $x(t)$, for random fatigue loading history, the following time series representation is proposed:

$$x(t) = m(t) + s(t) \cdot n(t)$$

where $m(t)$ and $s(t)$ represent the nonstationary mean and time varying scaling function, respectively, while $n(t)$ is the stationary noise component with zero mean and unit variance. It is noted that the product $s(t) \cdot n(t)$ defines a random zero-mean process with nonstationary standard deviation (variance).

The objective of random fatigue loading history modeling is, therefore, to seek the appropriate time-varying functions for $m(t)$, $s(t)$, and $n(t)$ such that the resulting analytical time series $x(t)$ will produce fatigue relevant events similar to that produced by the original history $D(t)$. To simplify the analysis, it will be assumed that the variations in mean, $m(t)$, and in scaling function, $s(t)$, are of slowly-varying processes as compared to the variation in the random noise, $n(t)$. This assumption is very realistic, especially for ground vehicle loading histories. Consequently, both $m(t)$ and $s(t)$ will be considered as smooth and continuous functions. Furthermore, the slowly-varying scaling function $s(t)$ in this case can also be viewed as the instantaneous standard deviation of the process at time t.

Nonstationary Variation in Mean

Because the mean variation is considered as a slowly varying process, only a finite number N_m of terms in a Fourier series is needed for accurately describe $m(t)$. This can be expressed as

$$m(t) = \overline{m} + \sum_{k=1}^{N_m} [a_k \cos \omega_k t + b_k \sin \omega_k t] \qquad (1)$$

where $\overline{m}$ is the long-term constant mean, a_k and b_k are the kth Fourier cosine and sine coefficients, respectively, and $\omega_k = 2\pi k / N\Delta$ is the kth Fourier frequency. The number of Fourier harmonics, N_m, is determined such that the resulting sequence, $D'(t)$, formed by the difference between the original history, $D(t)$, and the truncated Fourier series, $m(t)$, becomes stationary with respect to mean variation.

Nonstationary Variation in Variance

The remaining zero-mean process

$$D'(t) = D(t) - m(t) \qquad (2)$$

represents nonstationary variation in variance of the loading. To model this variation, the scaling function, $s(t)$, is evaluated by fitting the time-varying standard deviation function, $s'(t)$, of the underlying process, $D'(t)$. The instantaneous standard deviation at time $t = k\Delta$, s'_k, can be calculated by considering the ensemble average of the sample histories. In the case when only one single loading history is available, s'_k can be approximated by taking the temporal average of the form

$$s'^2_k = \sum_{j=0}^{n} w_{(j-n/2)} \left(D'_{(k+j-n/2)} \right)^2 \qquad (3)$$

where $D'_j \equiv D'(t = j\Delta)$, w_j is the weighting factor for a triangular window defined by

$$w_j = \begin{cases} \dfrac{4j}{n^2} & \text{for } j \leq \dfrac{n}{2} \\ \dfrac{4}{n} - \dfrac{4j}{n^2} & \text{for } j > \dfrac{n}{2} \end{cases} \qquad (4)$$

and n is the number of points (window width) used in the averaging process.

Because standard deviation, s'_k, is a non-negative quantity, its distribution is generally skewed with respect to the average value. However, in order to minimize the number of parameters needed in modeling the scaling function $s(t)$, a symmetrically distributed sequence is highly desirable. To accomplish this, the Box-Cox power transformation [11] is applied. The transformation is defined by

$$(s'_{BC})_k = \begin{cases} (s'_k)^\lambda & \text{for } \lambda \neq 0 \\ \log s'_k & \text{for } \lambda = 0 \end{cases} \qquad (5)$$

where $(s'_{BC})_k$ is the Box-Cox transform of s'_k and λ is the transformation parameter. The parameter λ is chosen to minimize the skewness measure of the transformed sequence $(s'_{BC})_k$.

Similar to the case of nonstationary mean variation, the time-varying characteristics of the observed sequence $(s'_{BC})_k$ is modeled by a truncated Fourier series $s_{BC}(t)$ as

$$s_{BC}(t) = \overline{s}_{BC} + \sum_{k=1}^{N_s} [c_k \cos \omega_k t + d_k \sin \omega_k t] \qquad (6)$$

where $\overline{s}_{BC}$ is the long-term average, c_k and d_k are the kth Fourier cosine and sine coefficients, respectively. The number of required Fourier harmonics, N_s, is determined in a similar manner as N_m.

The corresponding scaling function $s(t)$ can now be readily obtained by taking the inverse Box-Cox transformation of $s_{BC}(t)$. It is recalled that this slowly-varying scaling function is also the instantaneous standard deviation function of the process.

Stationary Random Variation

Upon obtaining the mean, $m(t)$, and scaling, $s(t)$, functions, the remaining noise component

$$n'(t) = \frac{D(t) - m(t)}{s(t)} \quad (7)$$

becomes a stationary process with zero mean and unit variance. This process can be concisely and accurately described by an Autoregressive-Moving Average (ARMA) model, $n(t)$.

There are two components of an ARMA model: (1) the autoregressive part and (2) the moving average part. The autoregressive part characterizes the correlation relation between current observation of the output variable, $n_t \equiv n(t)$, and its own past observations, $n_{t-j} \equiv n(t - j\Delta)$, while the moving average part represents the dependence of the output process on the values of the input process, $a_{t-j} \equiv a(t - j\Delta)$. For example, the following is a pth order autoregressive and qth order moving average model, denoted as ARMA(p,q) [12]:

$$\begin{aligned} n_t - \phi_1 n_{t-1} - \phi_2 n_{t-2} - \ldots - \phi_p n_{t-p} = \\ a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q} \end{aligned} \quad (8)$$

where $\phi_i; i = 1, 2, \ldots, p$, and $\theta_i; i = 1, 2, \ldots, q$, are the autoregressive and moving average parameters, respectively. In the above model, the input $a(t)$ is assumed to be an independently and identically distributed random process, i.e., $a(t)$ itself is considered to be non-autoregressive.

The autoregressive moving average processes can also be conveniently expressed in the frequency domain. In this case, the one-sided power spectral density $W(f)$ corresponding to the ARMA(p,q) model is given by [12]

$$W(f) = 2\sigma_c^2 \frac{\left|1 - \sum_{k=1}^{q} \theta_k e^{-2ki\pi f}\right|}{\left|1 - \sum_{k=1}^{p} \phi_k e^{-2ki\pi f}\right|}; \quad 0 \le f \le \tfrac{1}{2} \quad (9)$$

where σ_a^2 is the variance of $a(t)$, f is the linear frequency, and $i^2 = -1$.

Nonparametric Statistics

In order to determine whether a sequence of observations is of random nature, statistical tests must be performed. If information about the distribution of the underlying sequence is not available, a nonparametric statistical tests is preferred. In nonparametric inference, the tests are based only on the relative occurrence of an event formed by the sequence. As a result, information and assumptions regarding the statistical distribution of the sequence are not required. In this paper, a method of nonparametric statistical analysis will be employed in determining the most appropriate values for N_m and N_s required by Eqs. (1) and (6). Specifically, three different run (sequence of a defined event) tests based on the total number of runs, the length of the longest run, and the number of runs up and down were chosen [13]. Depending on the required level of confidence α, different values for N_m and N_s are obtained. The detailed discussion concerning the tests will not be discussed here and is referred to [10].

Fatigue Life Analysis

When the local strain approach is employed in fatigue life analysis, it requires both the stable (half-life) cyclic stress-strain curve and the strain-life curve for the material. These are given by [14]

$$\epsilon_a = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{H'}\right)^{1/n'} \quad (10)$$

$$\epsilon_a = \frac{\sigma'_f}{E}(2N_0)^b + \epsilon'_f (2N_0)^c \quad (11)$$

where ϵ_a and σ_a are the strain and stress amplitudes, respectively, E is the elastic modulus, N_0 is the life in cycles for the case of zero-mean stress, and the parameters ϵ'_f, σ'_f, H', n', b, and c are material constants obtained from curve fitting the fatigue and stress-strain data.

The effect of mean stress $\overline{\sigma}$ on life may be estimated using the following relation [15]

$$N = N_0 \left(1 - \frac{\overline{\sigma}}{\sigma'_f}\right)^{-1/b} \quad (12)$$

where N is the life in cycles corresponding to the case of nonzero-mean stress, $\overline{\sigma}$.

According to the Palmgren-Miner rule, the damage D_{ij} that correspond to a particular rainflow cycle C_{ij}, which forms a closed local stress-strain hysteresis loop with means, $\overline{\sigma}_i$ and $\overline{\epsilon}_i$, and ranges, $\Delta\sigma_j$ and $\Delta\epsilon_j$, is calculated from the cycle ratio given by [14]

$$D_{ij} = \frac{n_{ij}}{N_{ij}} \quad (13)$$

where n_{ij} and N_{ij}; $i, j = 1, 2, \ldots, M$, are the number of cycles counted and the number of cycles to failure for the given rainflow cycle C_{ij}, respectively. The constant M is the number of class intervals used in the rainflow cycle counting.

The component life, N_B, is then calculated from the induced damage due to $\sum_{i=1}^{M}\sum_{j=1}^{M} n_{ij}$ rainflow cycles as

$$N_B = \left(\sum_{i=1}^{M} \sum_{j=i}^{M} D_{ij} \right)^{-1} \tag{14}$$

where N_B is given in terms of the number of blocks (repetitions) of the load history.

Results and Discussion

In this study, a typical block of nonstationary service loading history of a ground vehicle is chosen. It represents the history of strain gauge readings at a critical location while the vehicle is traveling on a rough road. The history contains $N = 10,240$ data points and is shown in Fig. 1a. It is noted that the nonstationary variation in mean is generally due to maneuvers, while the nonstationary variation in variance is induced by change in vehicle speed as well as road profile. Without loss of generality, the load history is normalized such that the overall mean is zero and the dimensionless standard deviation is equal to unity. The normalized minimum and maximum are found to be -3.921 and 2.754, respectively.

According to the proposed model (Eq. 1), the loading history is decomposed into three components: the nonstationary mean, $m(t)$, the nonstationary scaling function, $s(t)$, and the stationary random noise, $n(t)$. Both $m(t)$ and $s_{BC}(t)$ are modeled using the truncated Fourier series. The numbers of terms N_m and N_s needed for $m(t)$ and $s_{BC}(t)$ are determined via nonparametric statistical run tests. To describe the stationary random process $n(t)$, the Autoregressive Moving Average (ARMA) models are considered.

In order to obtain the Fourier coefficients for the mean function, $m(t)$, a discrete Fourier transform is performed on the original history, $D(t)$. This results in a total of $(N/2-1)$ harmonics. To model only the slowly-varying components, a finite number N_m = 41of low frequency components is considered. The plot of $m(t)$ is shown in Fig. 1b.

The observed time-varying standard deviation function $s'(t)$ can be obtained next by considering the mean removed sequence $D'(t)$, Fig. 1c. The instantaneous standard deviation s'_k at time $t = k\Delta$ is estimated according to Eq. (3) and the result is shown in Fig. 1d. The optimal value for λ in the Box-Cox transformation is found to be 0.403. The sequence of the transformed standard deviation $(s'_{BC})_k$ is plotted in Fig. 1e. It is obvious that the latter sequence is more evenly distributed about the average value than the former one. Next, the Fourier series representation, $s_{BC}(t)$, of the Box-Cox transformed sequence is modeled. For this case, the number of terms $N_s = 70$ in the Fourier series is found to be adequate. The plot of the corresponding time series is shown in Fig. 2a. The required scaling function, $s(t)$, is readily obtained by taking the inverse Box-Cox transformation of $s_{BC}(t)$ and is shown in Fig. 2b.

The remaining stationary zero-mean noise component $n'(t)$, Eq. (7), is to be represented by $n(t)$, an ARMA model. By comparing the correlation coefficients, ρ, between the power spectral densities of the sequence $n'(t)$ and of various ARMA(p,q) models, it is concluded that ARMA(8,0) with $\rho = 0.96$ is the optimal model. The two processes $n'(t)$ and $n(t)$ are plotted in Figs. 2c and 2d, respectively. Figure 2e shows the load history, $x(t) = m(t) + s(t) \cdot n(t)$.

Plots showing the original loading history and its typical reconstructions are given in Fig. 3. Comparison of the power spectral densities for the original and reconstructed histories is shown in Fig. 4.

For the purpose of fatigue life analysis, the local strain approach is employed. Fatigue life calculations subjected to both the original and reconstructed strain loading histories are presented for unnotched SAE 1045 steel [16] under uniaxial loading. In order to eliminate any bias introduced by a particular reconstruction, the average fatigue life obtained from a total of 64 independent reconstructed loading histories is considered. Figure 5 shows the strain life curves [17] corresponding to reconstructed loading histories with various ARMA models together with the upper and lower bounds [18] obtained from the original loading history. A fairly good agreement is seen when the random noise is represented by an ARMA(8,0) model. This is noted by the corresponding life curve falls within the upper and lower bounds predicted by the original history. The shortest life is found when employing ARMA(0,0) model while, on the other hand, the longest life is predicted when using ARMA(1,0) model.

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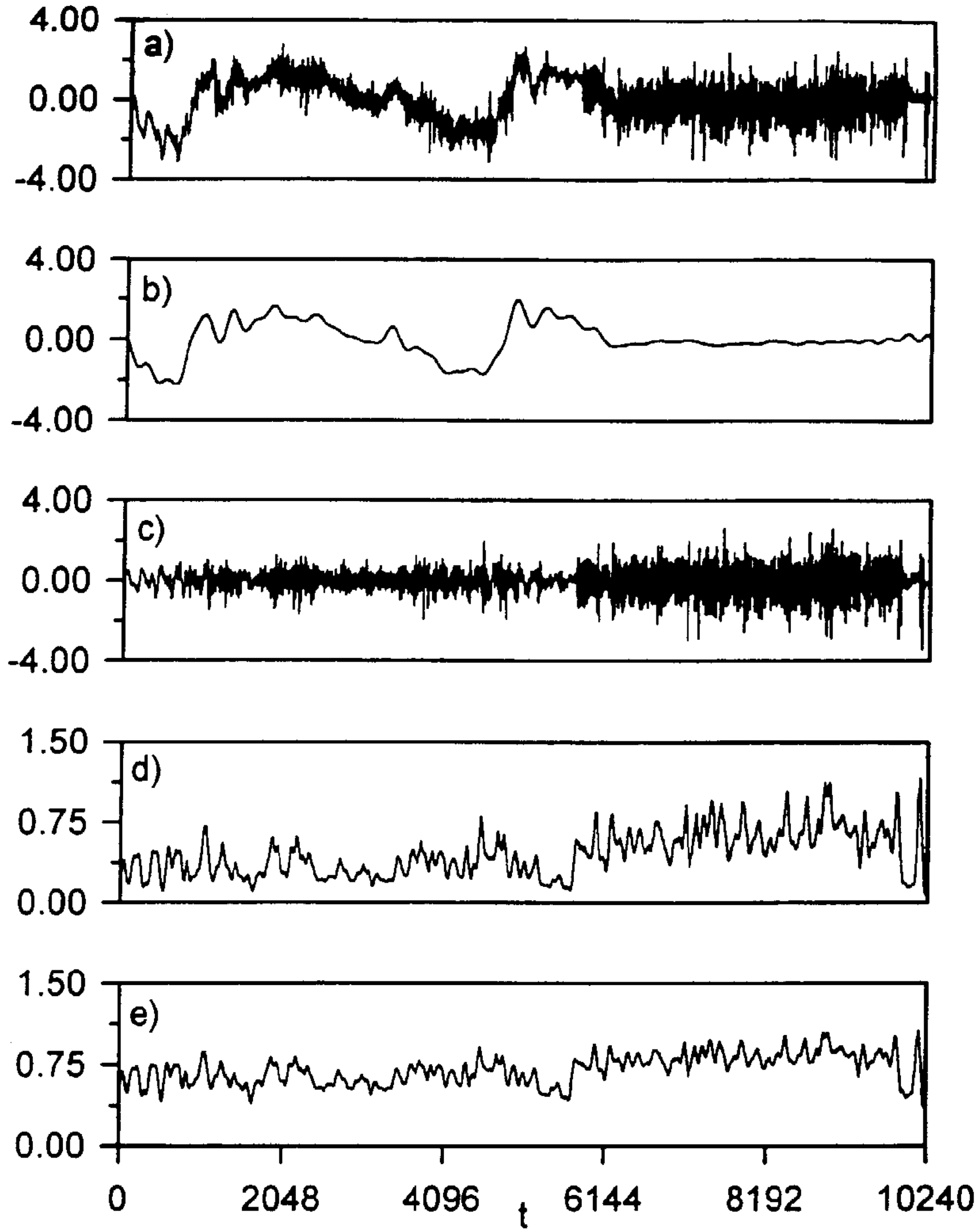

Fig. 1 Time series plots for a) original history, $D(t)$, b) mean variation, $m(t)$, c) mean-removed process, $D'(t)$, d) standard deviation (scaling) function for $D'(t)$, $s'(t)$, and e) Box-Cox transformation of $s'(t)$, $s'_{BC}(t)$.

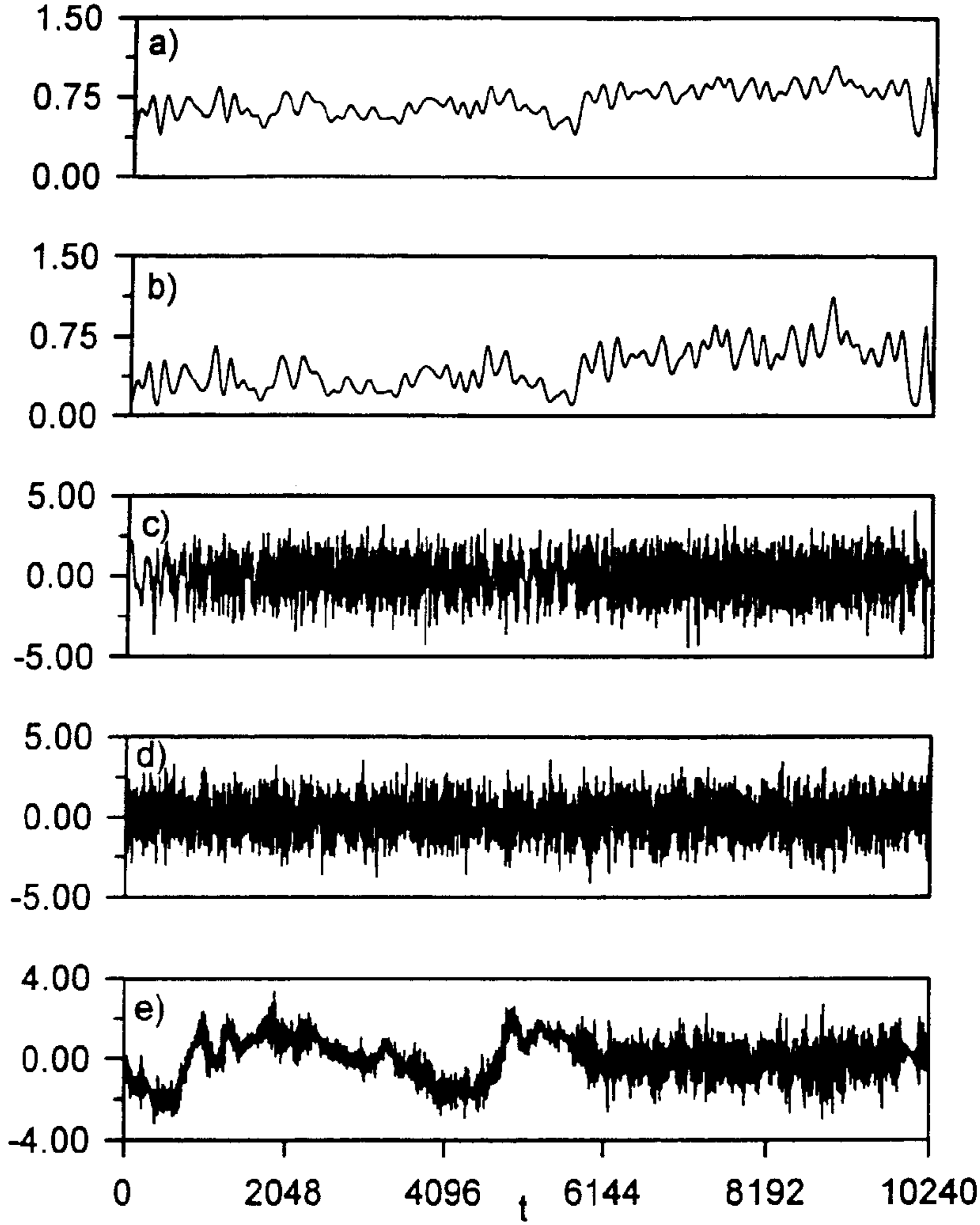

Fig. 2 Time series plots for a) Fourier representation of $s'_{BC}(t)$, $s_{BC}(t)$, b) analytical scaling function, $s(t)$, c) stationary noise component, $n'(t)$, d) ARMA(8,0) model, $n(t)$, and e) reconstruction of the loading history, $x(t)$.

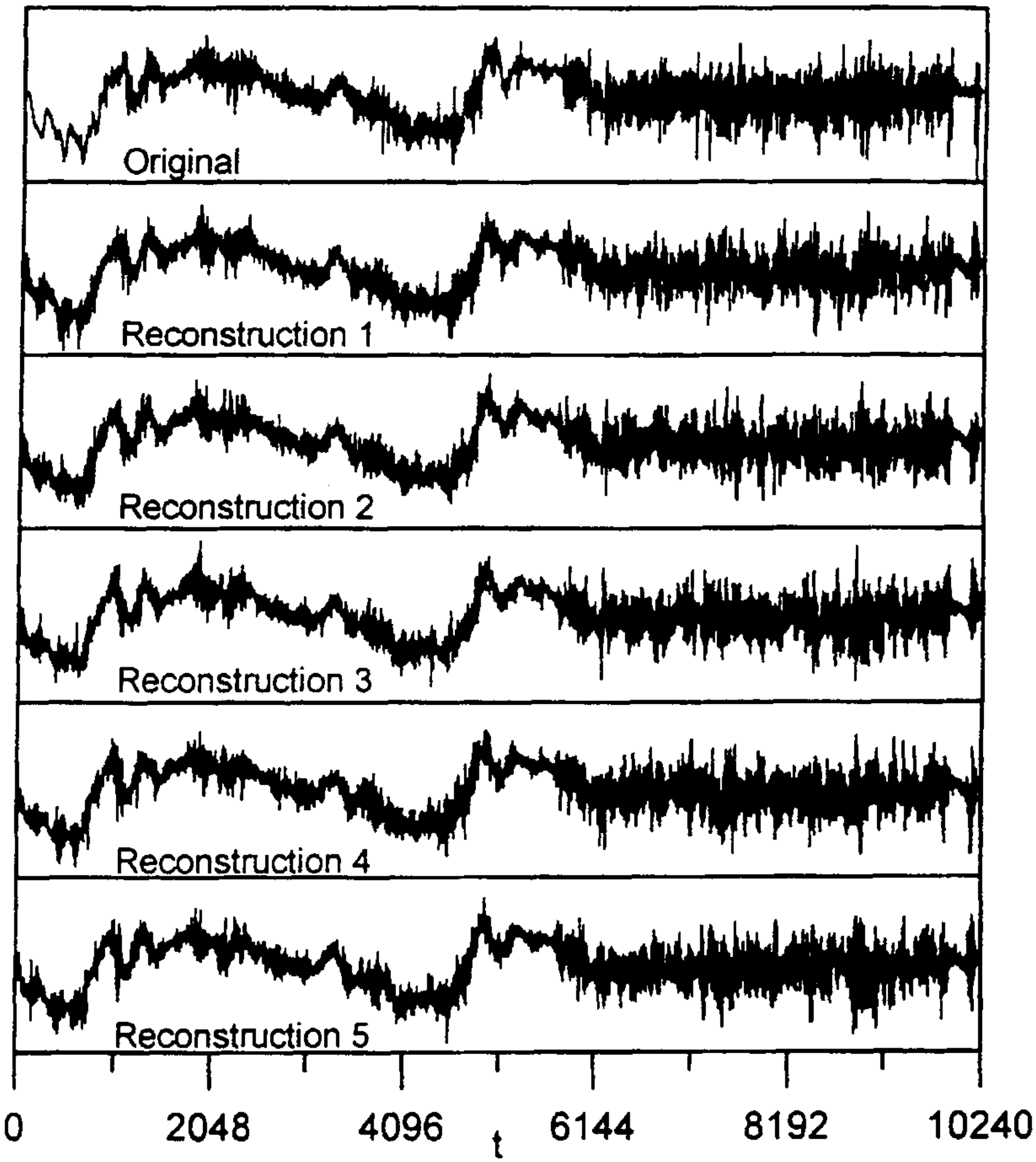

Fig. 3 Time series plots for original, $D(t)$, and various reconstructed, $x(t)$, loading histories.

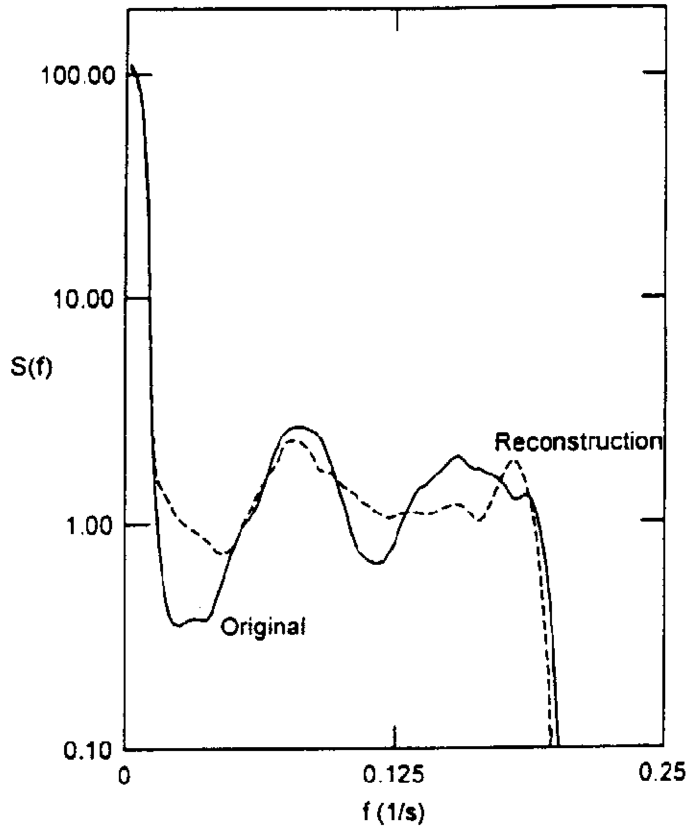

Fig. 4 Power spectral densities for original, $D(t)$, and reconstructed, $x(t)$, loading histories.

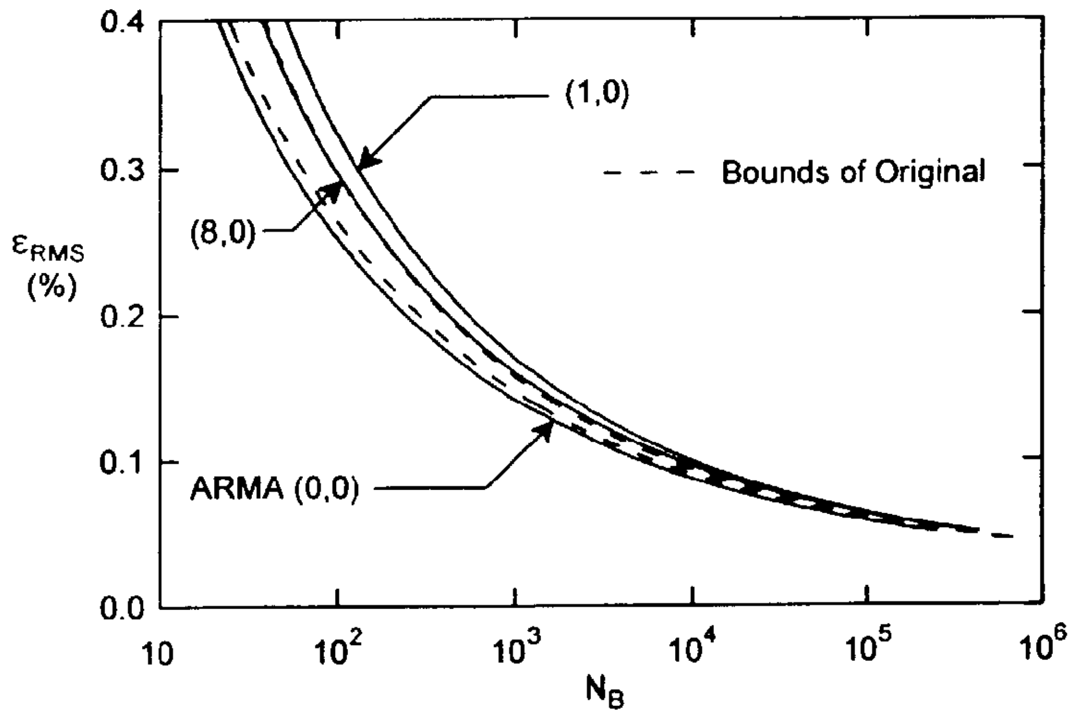

Fig. 5 Fatigue life curves (number of blocks to failure, N_B, vs. strain root-mean-square, ϵ_{RMS}) for reconstructed loading histories with various ARMA models, together with the lower and upper bounds predicted by the original history.