Modeling of non-stationary variance in vehicle loading histories for fatigue analysis

Fatigue 2000
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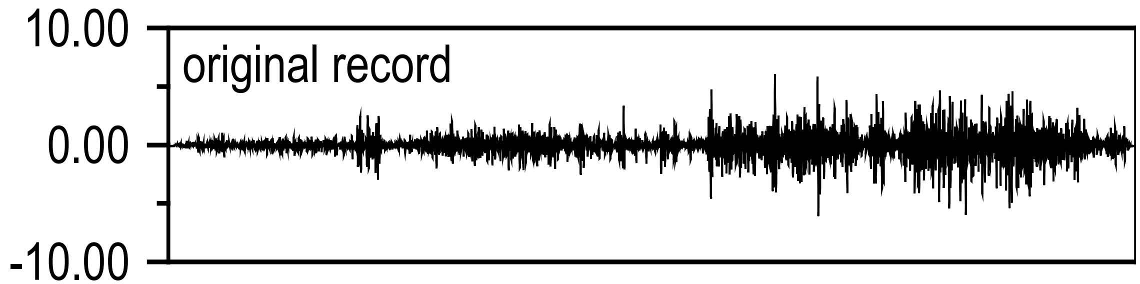
Concise fatigue load description



$$\mathbf{x}_t = \mathbf{s}_t \cdot \mathbf{n}_t$$

Motivation:

- storage reduction
- test machine / FEA compatibility
- monitoring
- concentration on relevant content
- elimination of non damaging events
- manipulation
  superposition
  extrapolation

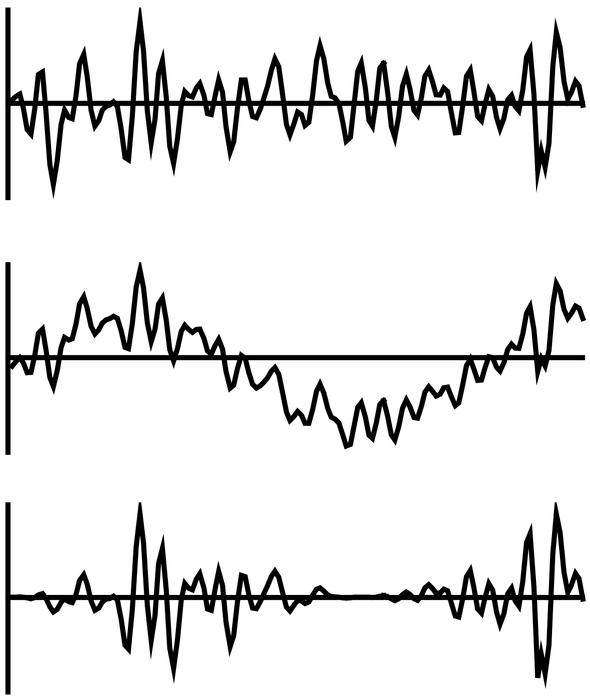
General Random Fatigue Loads

- Stationary
- Nonstationary

  in mean

  and / or

  in variance



Model for Nonstationary Fatigue Loading:

x(t) = s(t) n(t)

s(t) … variance scaling function

$$s_t^{BC} = \frac{1}{2}c_0 + \sum_{k=1}^{M_s}\left[c_k \cos\left(\omega_0 k t \Delta t\right) + d_k \sin\left(\omega_0 k t \Delta t\right)\right]$$

n(t) … zero mean, stationary noise content

ARMA(p,q)

Data Reduction Schemes

full time record $\Leftrightarrow$ Counting Methods (Markov - Rainflow)

$\Downarrow$

time domain $\Leftrightarrow$ frequency domain

$\Downarrow$

stochastic $\Leftrightarrow$ deterministic

$\Downarrow$

stationary $\Leftrightarrow$ non-stationary

$\Downarrow$

general linear model $\Leftrightarrow$ non-linear models

$\Downarrow$

ARMA model

$$n_t - \phi_1 n_{t-1} - \ldots - \phi_p n_{t-p} = e_t - \theta_1 e_{t-1} - \ldots \theta_q e_{t-q}$$

Characteristics of ARMA Method

Modeled		*Not Modeled*
Dynamics		Rainflow Cycles
Auto-Cross-Correlations		Extremes

Fundamental

Advantages	*Disadvantages*
Stochastic Description (Ensemble)	"Involved" Model Building
Consistent Multichannel Extension	Not Same Rainflow Cycles
Continuous Nonstationarity Description	Not Identical Extremes
Concise Description	
Simple, Fast Simulation	

Fatigue Life Calculation

•RMS scaling fatcors

•Rainflow Count

•Local Strain Approach

•Palmgren Miner Rule

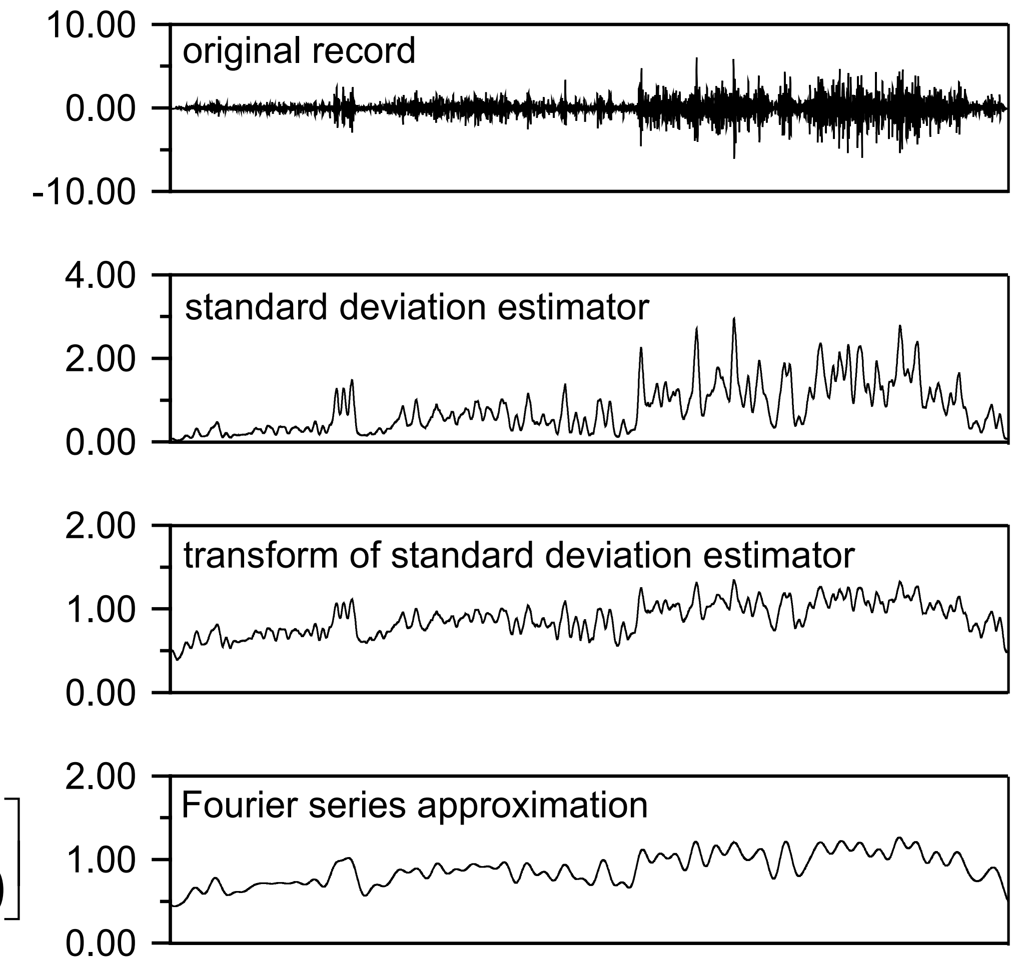
Reconstruction Procedure

$$x_t = s_t \cdot n_t$$

$$\tilde{\sigma}_t^2 = \sum_{j=0}^{n} w_{j-\frac{n}{2}} x_{t+j-\frac{n}{2}}^2$$

$$\tilde{\sigma}_t^{BC} = \begin{cases} \tilde{\sigma}_t^{\lambda} & \text{for} \quad \lambda \neq 0 \\ \log \tilde{\sigma}_t & \text{for} \quad \lambda = 0 \end{cases}$$

$$s_t^{BC} = \frac{1}{2} c_0 + \sum_{k=1}^{M} \begin{bmatrix} c_k \cos(\omega_0 kt\Delta t) \\ + d_k \sin(\omega_0 kt\Delta t) \end{bmatrix}$$



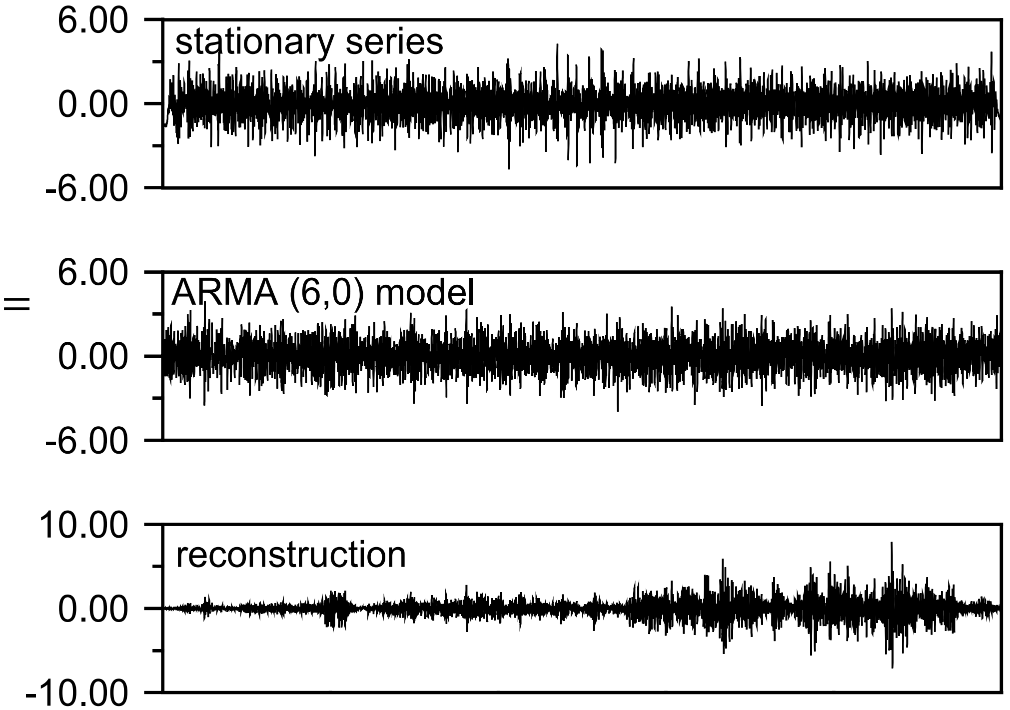
Reconstruction

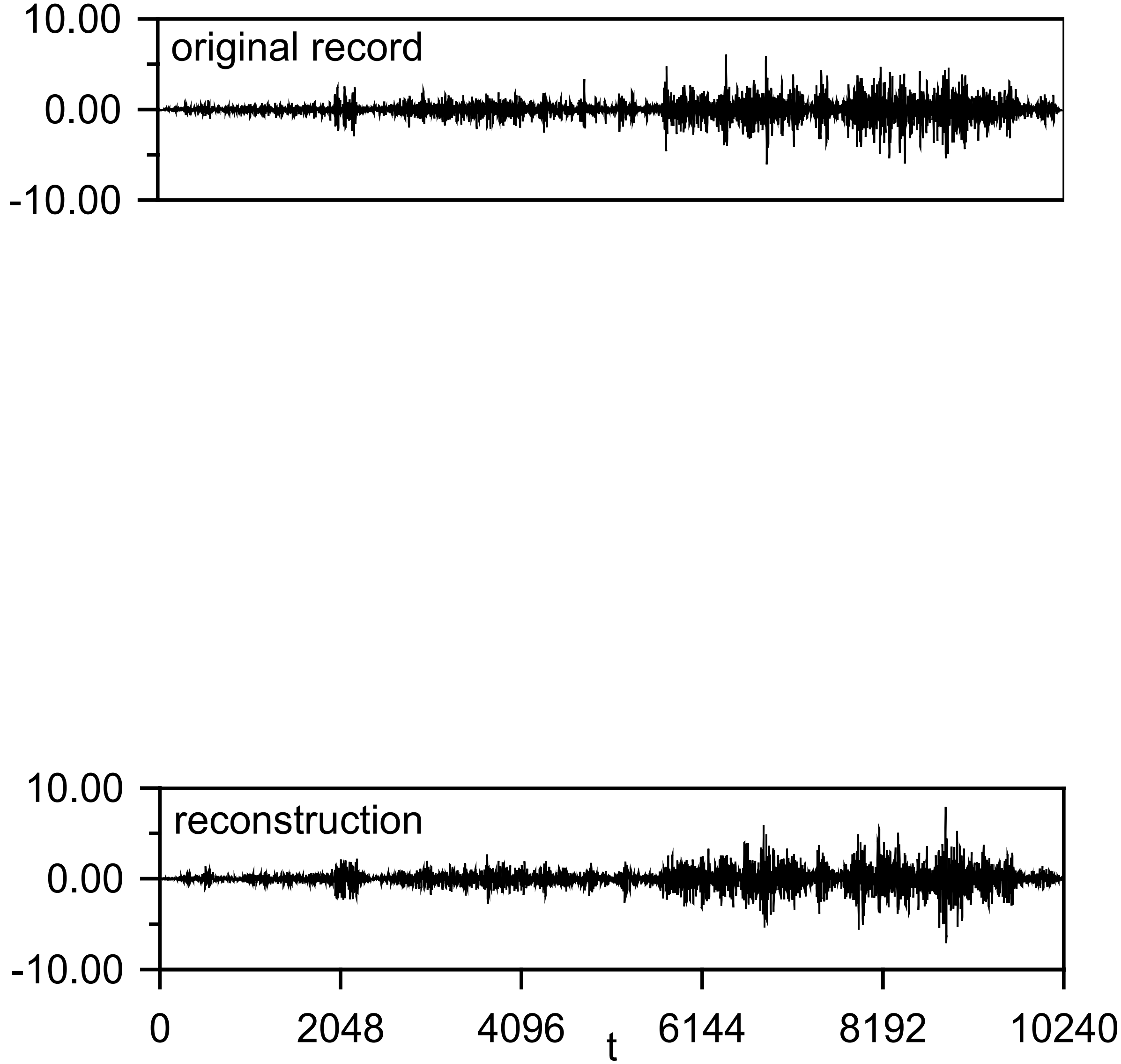
Procedure

$$x_t / \sigma_t = n_t$$

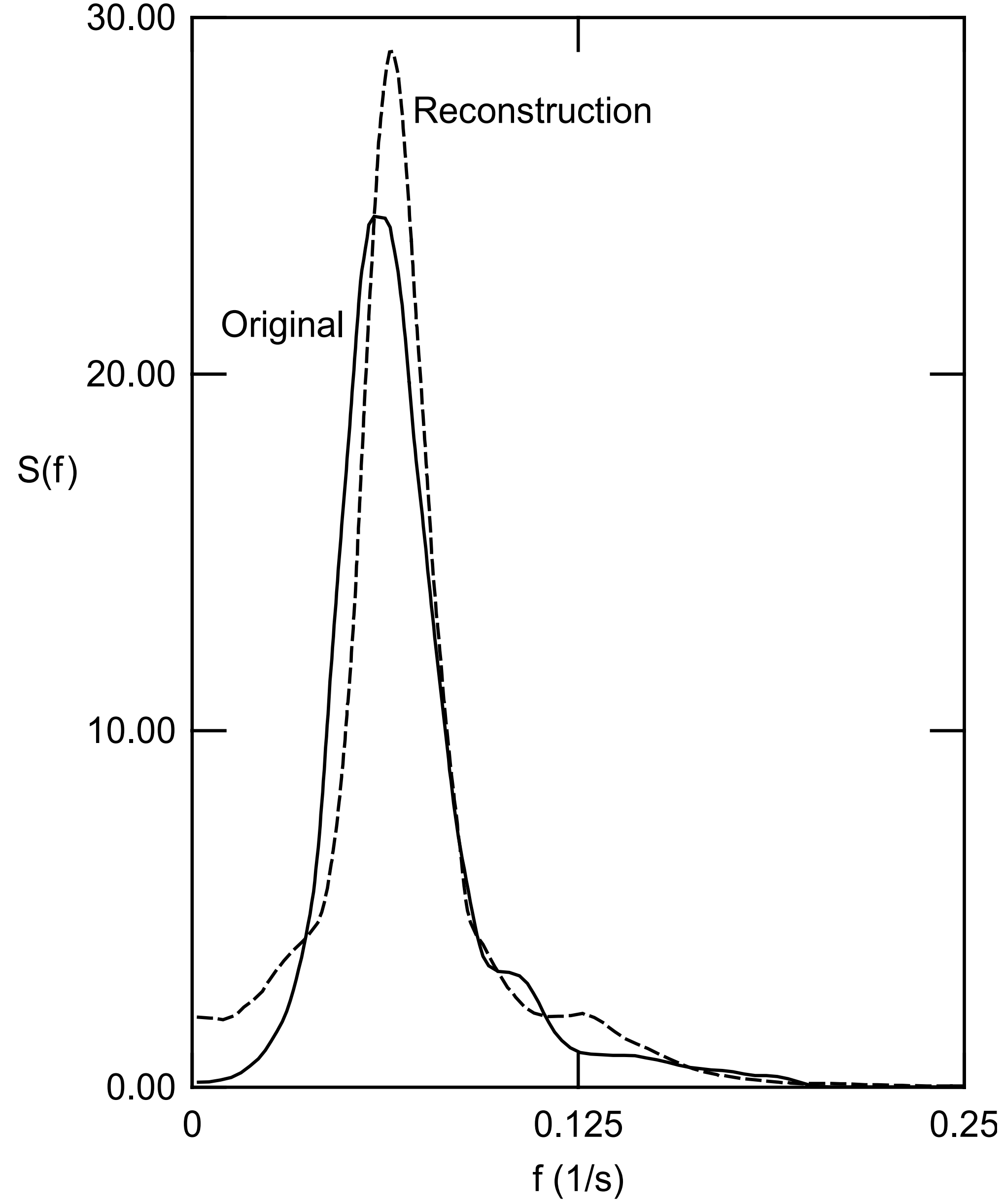
$$n_t - \phi_1 n_{t-1} - \phi_2 n_{t-2} - \ldots - \phi_p n_{t-p} =$$
$$a_t - \theta_1 a_{t-1} - \theta_2 a_{t-2} - \ldots - \theta_q a_{t-q}$$

$$x_t = s_t \cdot n_t$$

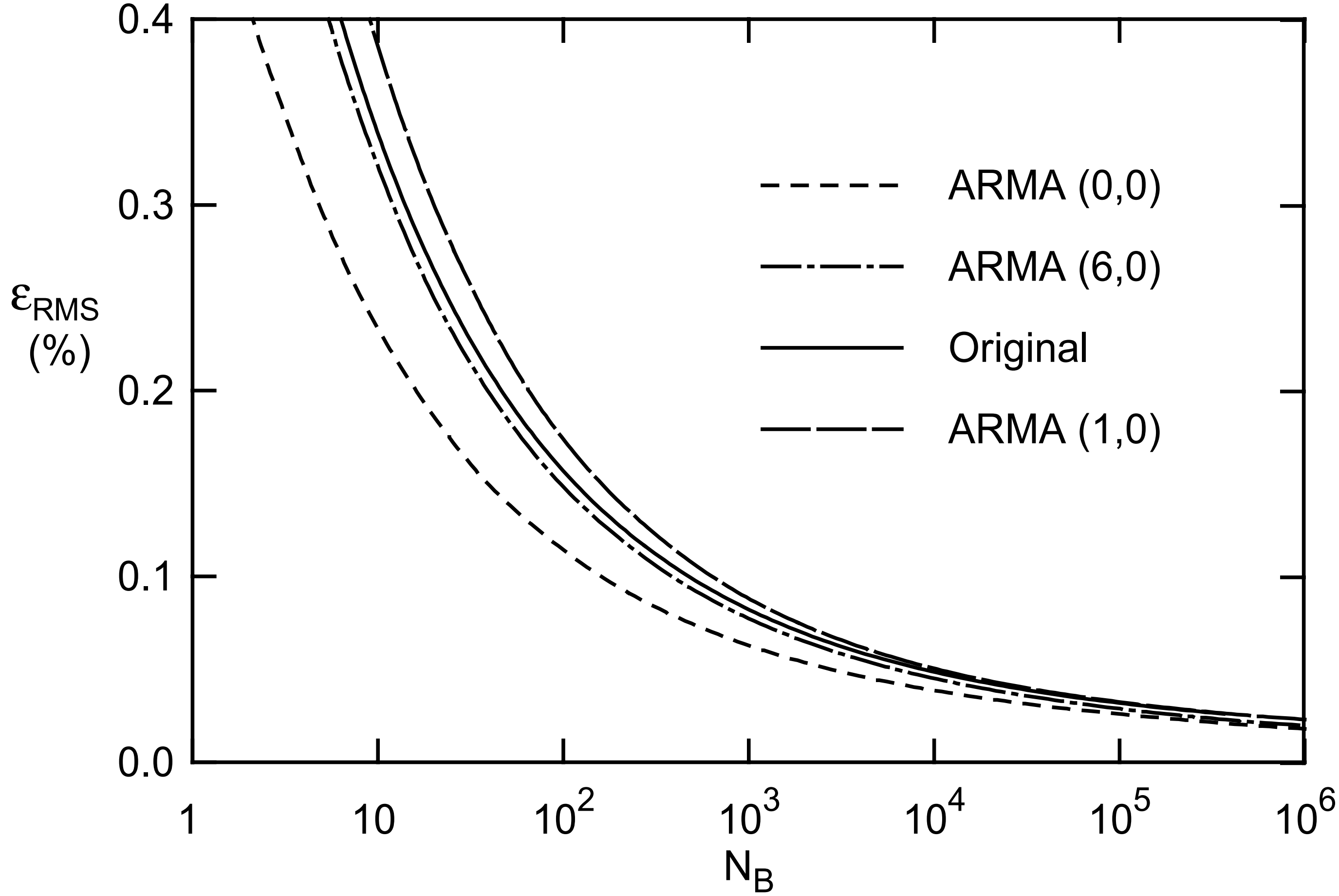



10.00
original record
0.00
-10.00
10.00
reconstruction
0.00
-10.00
0
2048
4096
t
6144
8192
10240

Auto Spectral Density



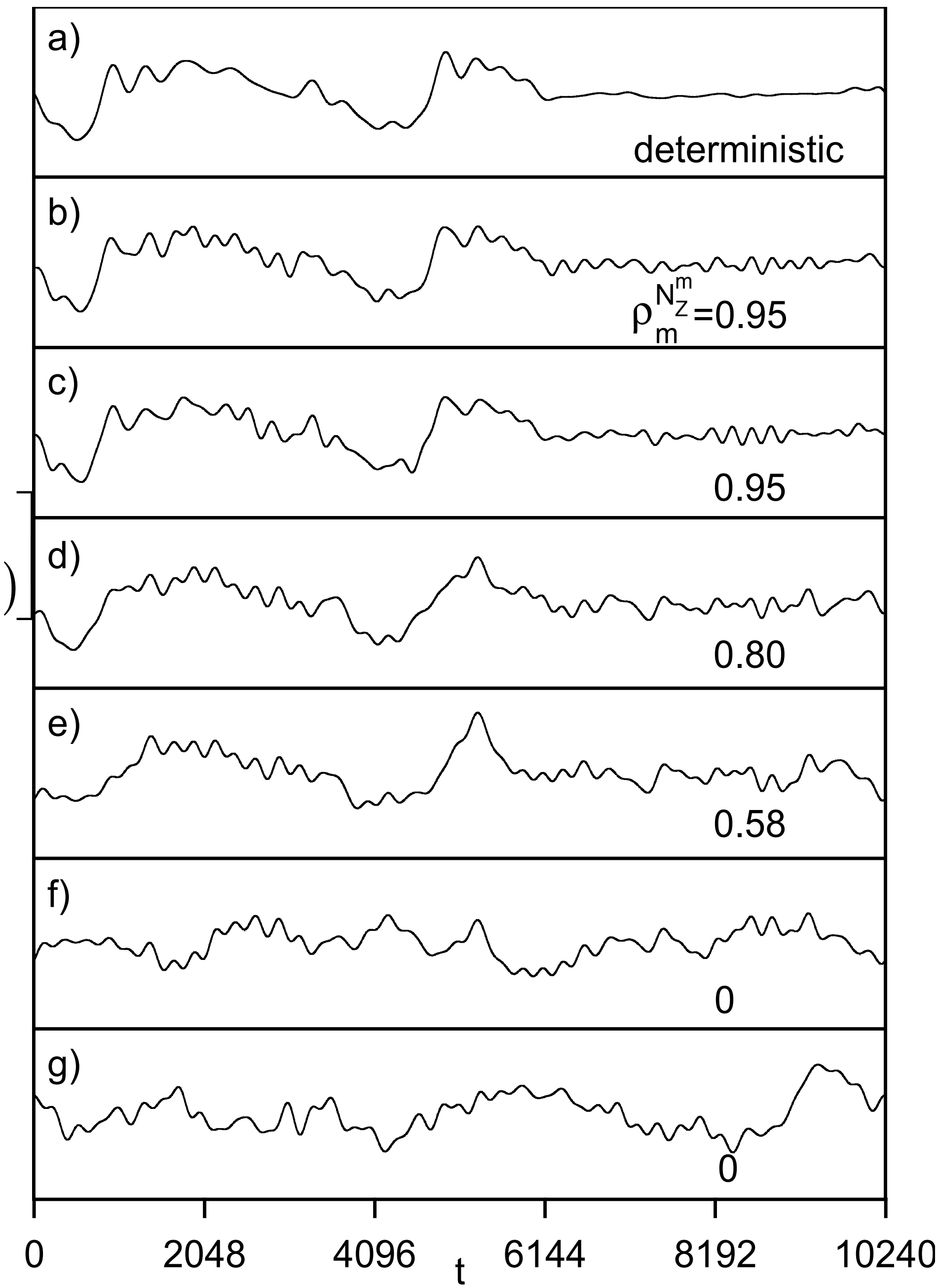
Fatigue Life



Ensemble Generation

$$s_t^{N_Z} = \frac{1}{2}c_0 + \sum_{k=1}^{M}\left[\begin{array}{l} c_k \cos(\omega_0 k t\Delta t - \gamma_k) \\ + d_k \sin(\omega_0 k t\Delta t - \delta_k) \end{array}\right]$$

$$\gamma_k, \delta_k = \begin{cases} U & \text{for} \quad k \in (N_Z + 1, M) \\ 0 & \text{for} \quad k \in (0, N_Z) \end{cases}$$



Questions Please ?