The Rainflow Method in Fatigue

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SOME COMMENTS ON METHODS OF REDUCING AND RECONSTRUCTING IRREGULAR FATIGUE LOADING HISTORIES

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Abstract: The methods of irregular fatigue loading history characterization and reconstruction are briefly reviewed for use in life prediction and efficient laboratory testing. The use of Autoregressive Moving Average (ARMA) models is introduced.

1. INTRODUCTION

For successful design against fatigue failure, an accurate method for life prediction as well as an efficient procedure for laboratory testing are required. To what extent these tasks can be accomplished, in turn, depends strongly on how accurately the expected loads are being described. Due to the fact that general fatigue loading histories are lengthy and random in nature, the development of an accurate but nevertheless concise method for describing such histories is deemed neccessary. The method is required not only to be able to preserve all fatigue damage relevant events, but also to contain a minimum number of model parameters.

The methods of modeling irregular fatigue loading histories can be divided into two broad categories— one that models only the extreme values and the other that models the complete history [1]. A successful load model is required to produce reconstructed histories that have an equivalent number of closed stress-strain hysteresis loops and a similar sequence of loadings as the original history. Furthermore, the reconstruction procedure needs to be both concise and efficient. Five methods of fatigue loading history modeling are reviewed in this paper.

2. METHODS OF FATIGUE LOADING HISTORY MODELING

The methods of modeling only the extreme values discard all intermediate points that exist between peaks and valleys. Consequently, the number of data points can be greatly reduced. The Rainflow Matrix and To-From Matrix methods of fatigue loading history reconstruction fall in this category.

In the *Rainflow Matrix* method [2-6] of load history reconstruction, the fatigue relevant information is summarized in the form of a rainflow range-mean matrix containing the distribution of closed stress-strain hysteresis loops formed by the original history. The history is discretized by dividing the entire load range into a finite number of levels. As a result, the rainflow cycles of the reconstructed histories are identical to those of the original history. The reconstruction, however, yields histories with a different sequence of loadings. The fatigue lives of the original and reconstructed histories are, therefore, expected to be different, but the difference is usually modest. However, a simplified strain-based fatigue life calculation yielding only upper and lower bounds will give the same result for both the original history and the reconstruction. An extension to a three-dimensional matrix allows the incorporation of the relative time increment between the adjacent peak and valley to be made, but this dramatically increases numerical storage requirements.

The *To-From Matrix* method [7-9] of fatigue load reconstruction requires information concerning the transition behavior between adjacent peaks and valleys. Similar to the Rainflow Matrix method of reconstruction, the load history is discretized into a convenient number of levels. The time series for peaks and valleys is regenerated using the To-From Matrix without considering the intermediate points. As a result, this method provides an identical number of peaks and valleys as of the original history but results in a different sequence of loadings. Unlike the Rainflow Matrix method, the To-from Matrix method will not give the same upper and lower bounds on fatigue life as of the original history. If the original history is a stationary Gaussian process, then the original and reconstructed histories are statistically equivalent, implying statistically identical fatigue life.

While these two methods reconstruct the loading histories accurately with respect to the number and magnitude of peaks and valleys of the original history, neither one accounts for the intermediate points. That is, the frequency content of the original history is not preserved during the reconstruction. Since the methods of reconstructing the complete load history consider the transition characteristics of each individual point, they can be employed to preserve the frequency content of the original history. Furthermore, the methods can be performed in either the time or frequency domains. Of the frequency domain approaches, the principal method used is the Power Spectral Density. Random process theory is used for the time domain methods. These include the Markov method and a method using Autoregressive Moving Average processes.

In the frequency domain description of random fatigue loading histories, a method of *Power Spectral Density* (PSD) is generally employed [10-12]. The Fast Fourier transform technique can be efficiently used to obtain the power spectral density of the original history. The reconstruction procedure involves a discrete inverse Fourier transform coupled together with the method of random phase angles. In this part of the study, emphasis was given to accurate reproduction of frequency content of the original history. Therefore, no averaging of periodogram amplitudes was performed, which implies retaining all terms of the Fourier transform of the original record. A comment with regard to periodicity of the regenerated history seems in order to clarify the current literature on this topic. Regardless of the details of reconstruction, a periodic time series will be obtained, rendering this method not truly stochastic. By using an analytical expression to represent the spectral density, the original history can be concisely described and the reconstruction can be efficiently performed via a limited number of parameters. In this approach, the frequency content is preserved while the number and magnitude of closed stress-strain hysteresis loops is not identical to that of the original history.

Figure 1 shows the original and two typical rainflow reconstructed fatigue loading histories, and also a PSD and a To-From regeneration.

The *Markov* method of fatigue load history reconstruction [13] is based on a random process that has a single step memory, i.e. the current value of the process depends only on the previous value. The load can be modeled either as a discrete or continuous random variable. Transition probabilities for any two adjacent points are deduced from the original history. The Markov method is particular successful for processes which only contain correlation between two adjacent points. Unlike the To-From Matrix method, this method allows the generation of intermediate points between the extremes.

A class of time series models referred to as *Autoregressive Moving Average* (ARMA) has recently been found in an increasing number of applications in a wide range of practical engineering problems. In the ARMA models, the correlation or dependence among the observations is expressed in the form of linear stochastic difference equations of various orders. There are two parts of an ARMA model: (a) the autoregressive part and (b) the moving average part. The autoregressive part represents the correlation of the variable with its own past, while the moving average part expresses the dependence of the variable on the past and present values of a random disturbance. It is noted that the ARMA(1,0) process is equivalent to the Markov process.

Limited application of ARMA processes for representing random fatigue loading histories has recently been undertaken [14]. It is found that the models can be used to provide an effective means for reconstruction of some particular load histories. As compared to other history reconstruction technigues, such as the Rainflow Matrix, To-From Matrix, and PSD, ARMA models require many fewer parameters to accurately describe the behavior of a process. As a result, the reconstruction of an original history by ARMA models for computer simulation and laboratory testing can be accomplished in a relatively fast and straigthforward manner. Furthermore, since not only the peaks and valleys but all the adjacent data points are regenerated, the frequency characteristics of the load histories are preserved.

Figure 2 compares an original fatigue loading history and corresponding typical reconstructions by ARMA models of different order. The resulting distributions of rainflow cycles for the original history and various ARMA reconstructions are shown in Figure 3.

For better judgement of successful regeneration, fatigue life was analyzed for the original history and ARMA models. The original loading history is a strain record measured during a field test. An analytical life analysis for an unnotched axially loaded member made of SAE 1045 steel was performed. The analysis follows the hysteresis loops formed during loading, taking into account mean stress effects. The resulting damage distribution according to rainflow cycles for the original history and ARMA(0,0), ARMA(1,0), ARMA(3,1), and ARMA(3,2) are shown in Figure 4. Strain life curves were obtained with the root mean square (RMS) values of strain ranging from 0.1% to 0.6%. The life is measured as the number of blocks to initiation of fatigue cracks. These results are shown in Figure 5.

Power spectral densities were also compared, and representative curves are shown in Figure 6. Models ARMA(2,1) and lower deviated somewhat from the original history, but models (3,1) and (3,2) gave very similar results to the original history.

3. DISCUSSION

The original data, when reduced to a rainflow cycle distribution, indicates a generally smooth distribution of cycles around the zero mean level as would be expected from the data source, a vehicle operating on a cobblestone track. For the ARMA(0,0) sequence the distribution shows many more cycles identified. The ARMA(0,0) model does not include parameters that reflect the correlation of points in the original history and is in effect a Gaussian white noise process. Consequently, for the same number of points in the time history, there are more turning points, and consequently more rainflow cycles. In addition the amplitude distribution is skewed toward small cycles.

Increasing the number of correlation terms used to describe the signal in the ARMA model improves the appearance of the amplitude distribution when compared with the original data. ARMA(3,1) shows good correlation between the original and reconstructed history cycle count distributions. Similar trends are observed when the cycle count distributions are used to calculate fatigue lives, shown in Table 1. Models of higher order also show good correlation with the original data, however the improvement over the simpler ARMA(3,1) model is small, as shown by the ARMA(3,2) model.

Although the ARMA models do not regenerate an exact distribution of rainflow cycles, the distribution of cycles is very similar to the original data. The accuracy with which the reconstruction technique reproduces cyclic events should be at least as good as the variation that one would observe in making multiple measurements of the same service event.

If amplitude distribution was the only criterion of interest, the rainflow reconstruction technique, which preserves exactly the rainflow cycle count would be clearly superior. Criteria other than amplitude distribution may also be of significance for some test and analysis applications. Of importance are the numerical complexity of regeneration, the ability to preserve frequency content as well as amplitude content, and the ability to be extended to regeneration of multichannel loadings.

4. CONCLUDING REMARKS

Depending on the objective of load reconstruction, each of the methods have their merits and limitations. The Rainflow Matrix and To-From Matrix methods reconstruct a time history of peaks and valleys and, therefore, provide sufficient information for single channel fatigue testing. They present concise and simple reconstruction schemes and are widely used. In the case of multiaxial fatigue loading, however, load histories need to preserve the frequency component of all load axes with respect to each other. The PSD, Markov, and ARMA methods generate histories with characteristics in both time and frequency domains preserved, therefore allowing for multichannel modeling. In addition, these three methods preserve, in a statistical sense, the sequence of events, while the first two methods ignore them. The ARMA processes possess an attractive degree of generality and, in contrast to the PSD method, preserves the time base of the original history, which facilitates modeling nonstationary loading where the mean level and variance may both change with time. Also, ARMA models in a sense incorporate the PSD method, as any ARMA model has a known PSD fixed by its parameters and available from a relatively simple analytical expression.

5. REFERENCES

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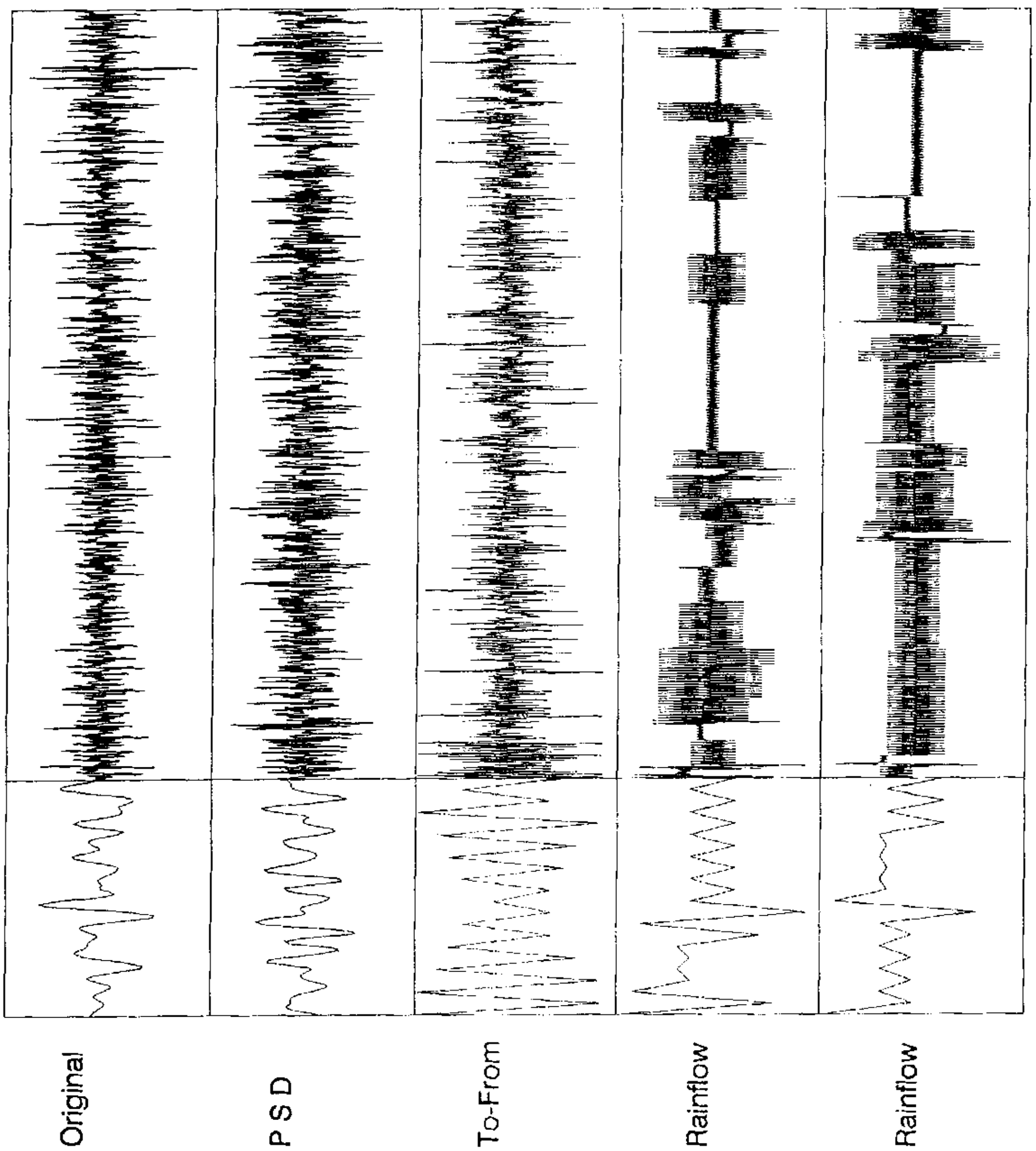

Figure 1. Original fatigue loading history and typical reconstructions by PSD, To-From, and Rainflow method

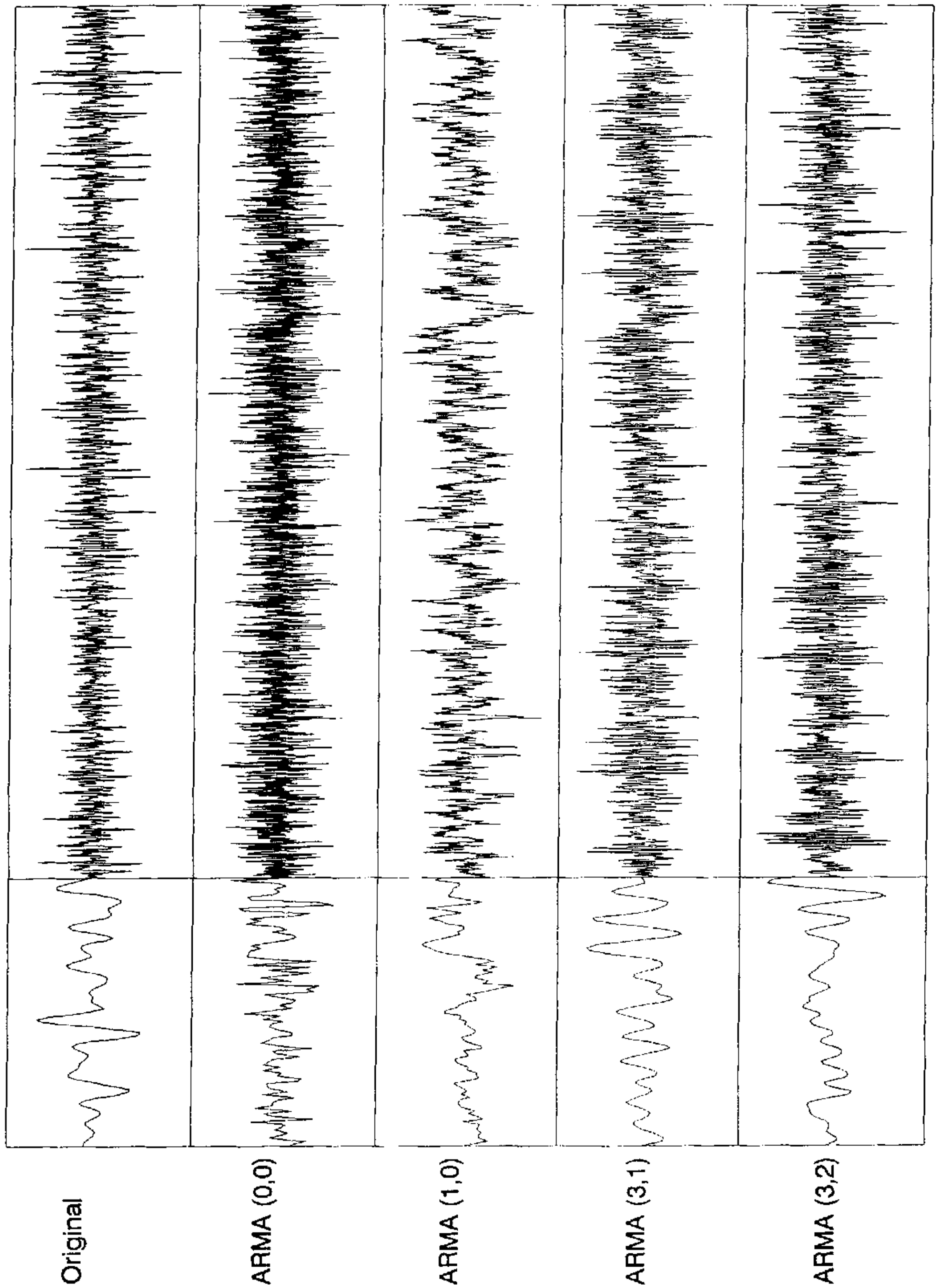

Figure 2. Original fatigue loading history and typical reconstructions by ARMA(0,0), ARMA(1,0), ARMA(3,1), and ARMA(3,2) models

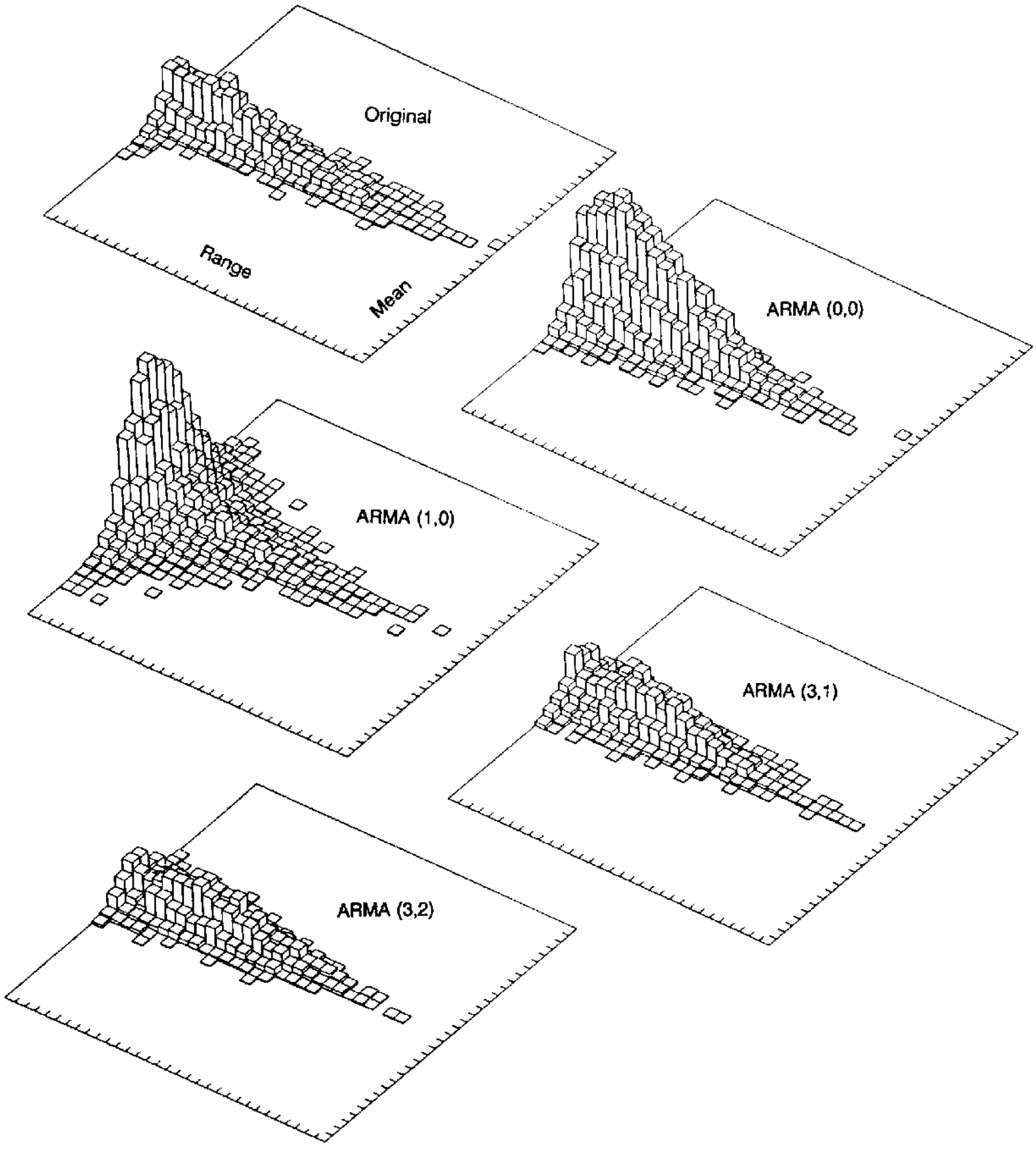

Figure 3. Distributions of rainflow cycles for original history and ARMA(0,0), ARMA(1,0), ARMA(2,1), and ARMA(3,1) reconstructions, all plotted at the same scale

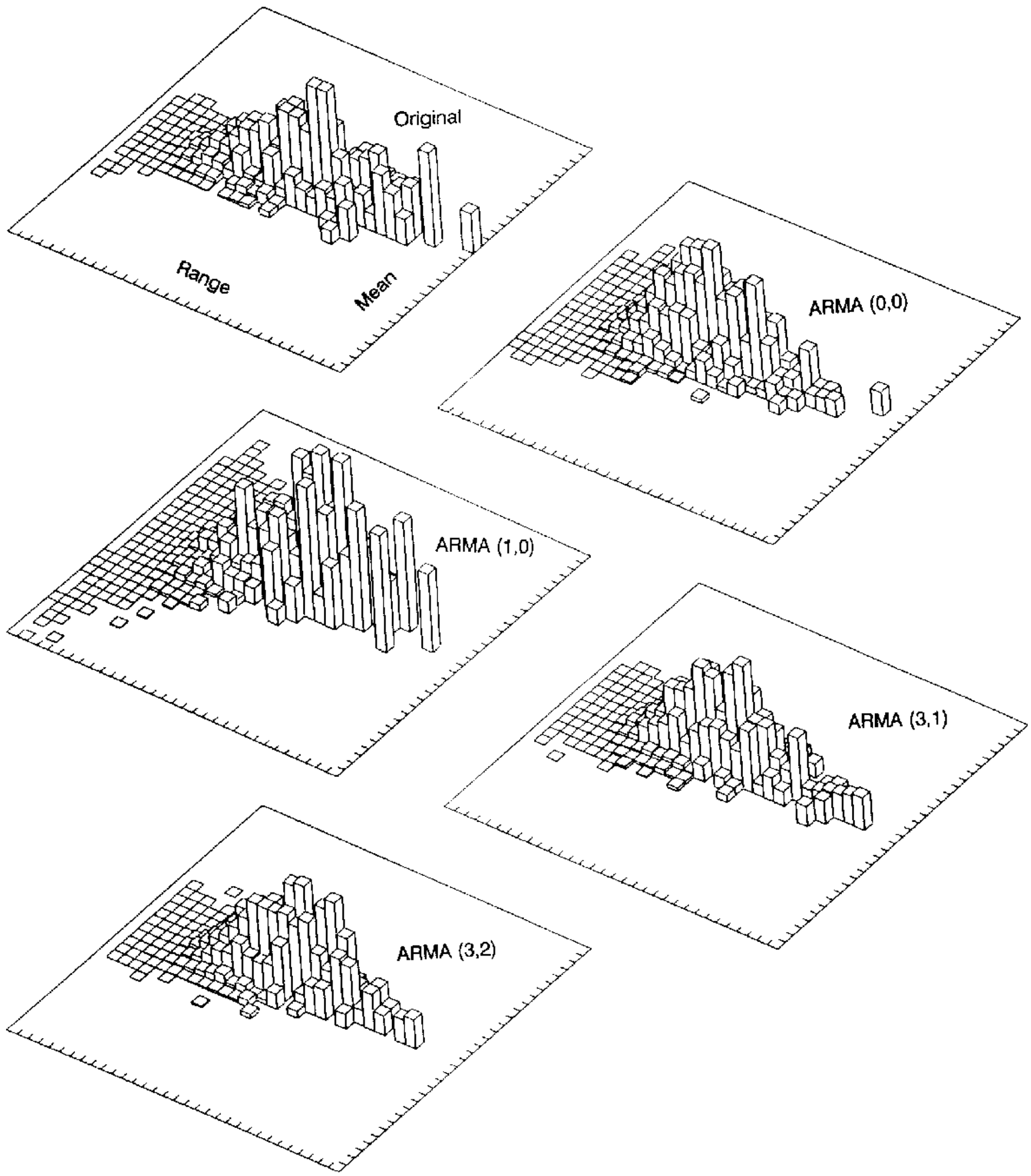

Figure 4. Distributions of damage for original history and ARMA(0,0), ARMA(1,0), ARMA(3,1), and ARMA(3,2) for RMS strain = 0.2%, all plotted at the same scale

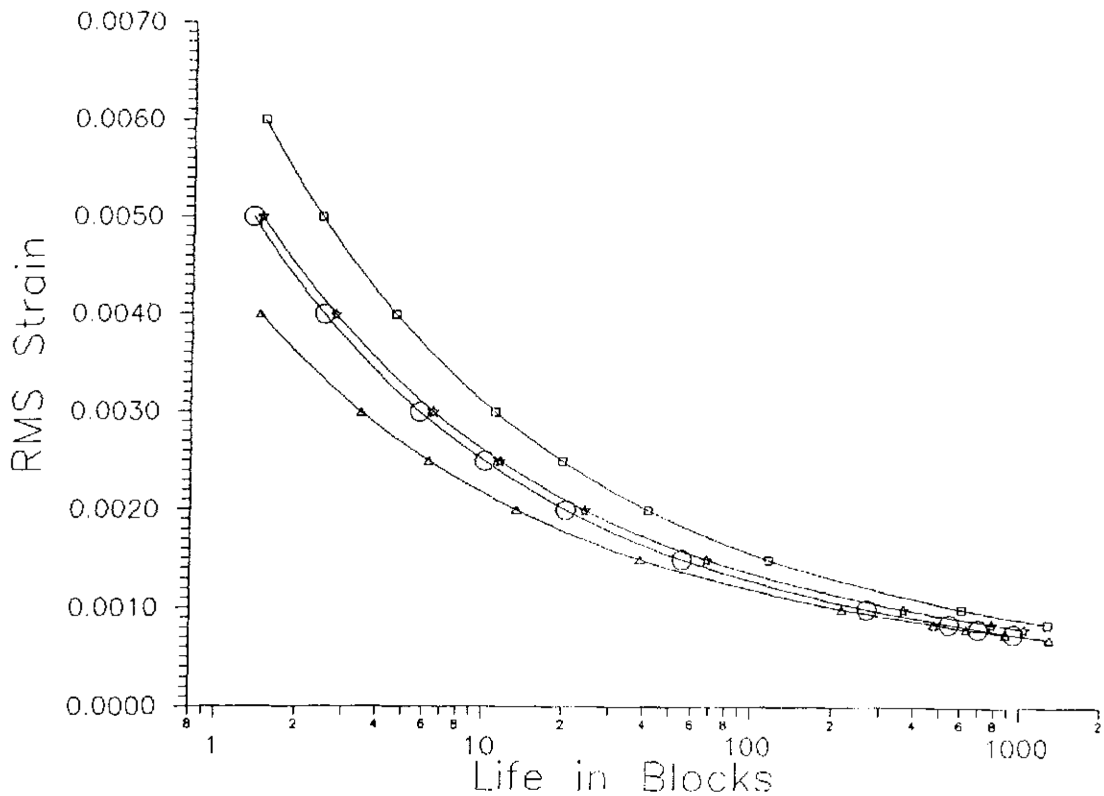

Figure 5. Predicted Strain - Life curves for the original history (circles) and ARMA(0,0) (triangles), ARMA(1,0) (squares), ARMA(3,1) (stars), and ARMA(3,2) (same as ARMA(3,1) reconstructions

Method of Regeneration	Life at RMS Strain = 0.2 %	Strain Range in % (10^{-2})	Number of Rainflow Cycles	Storage Required
Original Data	20.4	(-95 / 79)	2,547	22,450
ARMA (0,0)	13.3	(-91 / 77)	7,466	1
ARMA (1,0)	41.3	(-80 / 79)	5,873	2
ARMA (3,1)	24	(-74 / 80)	2,553	5
ARMA (3,2)	24	(-80 / 80)	2,577	6
P S D	24.1	(-73 / 80)	2,593	11,226
To-From	18	(-95 / 79)	2,367	32 x 32
Rainflow	20.2	(-95 / 79)	2,367	32 x 32

Table 1. Comparison of regeneration methods with respect to life, strain range, number of rainflow cycles, and storage requirement

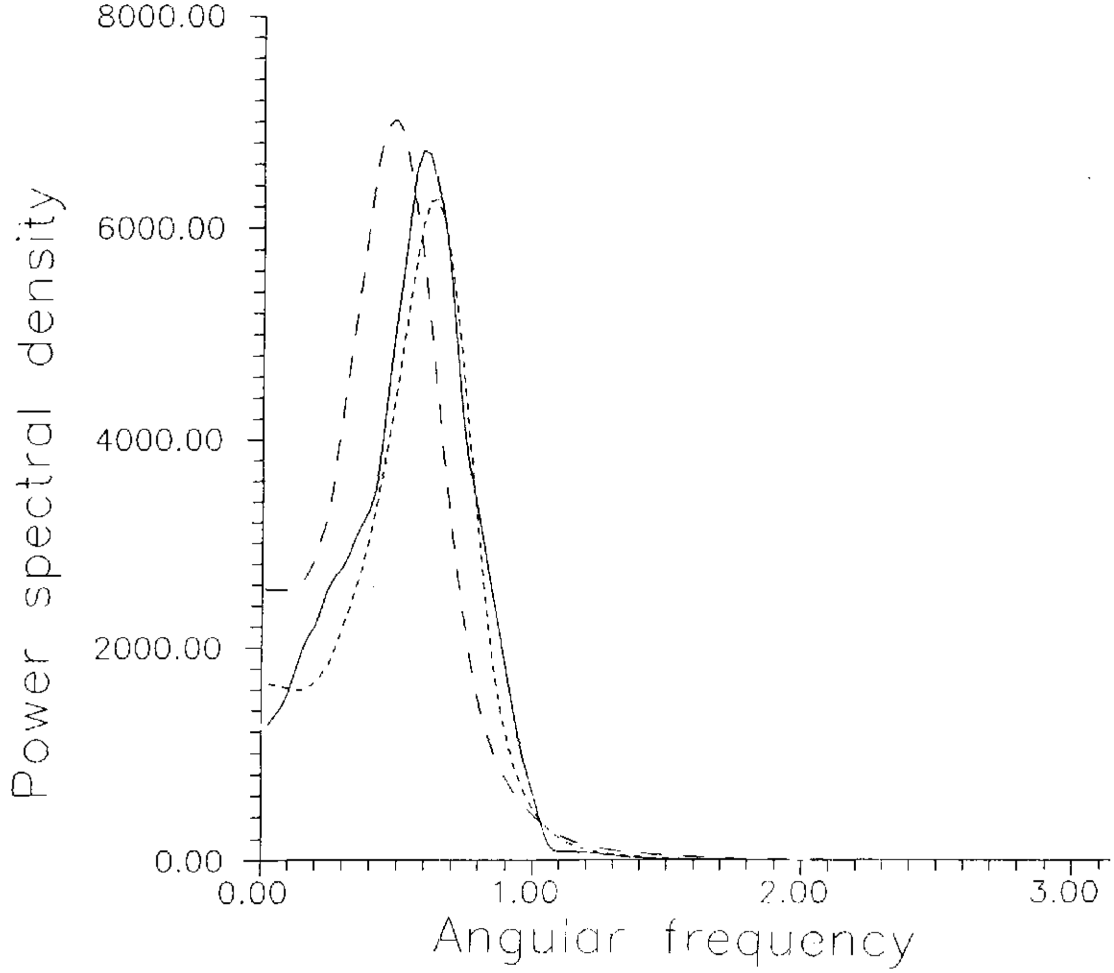

Figure 6. Power spectral density ($(in/in)^2$) for the original history (solid line), ARMA(2,1) (long dashes), and ARMA(3,2) (short dashes)